According to New Revised Credit System Syllabus

DBATU

Third Year (T.Y.) B. TECH Semester - VI

Course in ELECTRONICS & TELECOMMUNICATION ENGINEERING

ADVANCED DIGITAL SIGNAL PROCESSING



Dr. RAJ KISHORE PRASAD

www.pragationline.com
niralipune@pragationline.com
www.facebook.com/niralibooks
@nirali.prakashan

NIRALI PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

A TEXT BOOK OF

ADVANCED DIGITAL SIGNAL PROCESSING

(Elective – III)

FOR
SEMESTER – VI

THIRD YEAR (T.Y.) B. TECH COURSE IN ELECTRONICS AND TELE-COMMUNICATION ENGINEERING

Strictly According to New Revised Credit System Syllabus of Dr. Babasaheb Ambedkar Technological University (DBATU), Lonere, (Dist. Raigad) Maharashtra,
(w.e.f. June 2022-23)

Dr. RAJ KISHORE PRASAD
BE. (Honors), ME, MS, Ph.D
Ex. Head of Department
Department of E&TC Engineering,
Army Institute of Technology,
Pune 411 015.

NIRALI PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

N6811

ADVANCED DIGITAL SIGNAL PROCESSING (E&TC, DBATU) **ISBN : 978-81-19117-19-2**

First Edition : June 2023
© : Author

The text of this publication, or any part thereof, should not be reproduced or transmitted in any form or stored in any computer storage system or device for distribution including photocopy, recording, taping or information retrieval system or reproduced on any disc, tape, perforated media or other information storage device etc., without the written permission of Authors with whom the rights are reserved. Breach of this condition is liable for legal action.

Every effort has been made to avoid errors or omissions in this publication. In spite of this, errors may have crept in. Any mistake, error or discrepancy so noted and shall be brought to our notice shall be taken care of in the next edition. It is notified that neither the publisher nor the authors or seller shall be responsible for any damage or loss of action to any one, of any kind, in any manner, therefrom. The reader must cross check all the facts and contents with original Government notification or publications.

Published By :
NIRALI PRAKASHAN
Abhyudaya Pragati, 1312 Shivaji Nagar
Off J.M. Road, PUNE 411005
Tel : (020) 25512336/37/39
Email : niralipune@pragationline.com

Polyplate

Printed By :
YOGIRAJ PRINTERS AND BINDERS
Survey No. 10/1A, Ghule Industrial Estate
Nanded Gaon Road
Nanded, Pune 411041

DISTRIBUTION CENTRES

PUNE

Nirali Prakashan
(For orders outside Pune)
S. No. 28/27, Dhayari Narhe Road, Near Asian College
Pune 411041, Maharashtra
Tel : (020) 24690204; Mobile : 9657703143
Email : bookorder@pragationline.com

Nirali Prakashan
(For orders within Pune)
119, Budhwar Peth, Jogeshwari Mandir Lane
Pune 411002, Maharashtra
Mobile : 9657703145, 9890997937
Email : niralilocal@pragationline.com

MUMBAI

Nirali Prakashan
Rasdhara Co-op. Hsg. Society Ltd., 'D' Wing Ground Floor, 385 S.V.P. Road
Girgaum, Mumbai 400004, Maharashtra
Mobile : 7045821020, Tel : (022) 2385 6339 / 2386 9976
Email : niralimumbai@pragationline.com

DISTRIBUTION BRANCHES

DELHI

Nirali Prakashan
Room No. 2 Ground Floor
4575/15 Omkar Tower, Agarwal Road
Darya Ganj, New Delhi 110002
Mobile : 9555778814/9818561840
Email : delhi@niralibooks.com

BENGALURU

Nirali Prakashan
Maitri Ground Floor, Jaya
Apartments, No. 99, 6th Cross, 6th
Main, Malleswaram, Bengaluru
560003 Karnataka; Mob : 9686821074
Email : bengaluru@niralibooks.com

NAGPUR

Nirali Prakashan
Above Maratha Mandir, Shop No. 3,
First Floor, Rani Jhanshi Square,
Sitabuldi Nagpur 440012 (MAH)
Tel : (0712) 254 7129
Email : nagpur@niralibooks.com

KOLHAPUR

Nirali Prakashan
438/2, Bhosale Plaza, Ground Floor,
Khasbag, Opp. Balgopal Talim,
Kolhapur 416 012 Maharashtra
Mob : 9850046155
Email : kolhapur@niralibooks.com

JALGAON

Nirali Prakashan
34, V. V. Golani Market, Navi Peth,
Jalgaon 425001, Maharashtra
Tel : (0257) 222 0395
Mob : 94234 91860
Email : jalgaon@niralibooks.com

SOLAPUR

Nirali Prakashan
R-158/2, Avanti Nagar, Near Golden
Gate, Pune Naka Chowk
Solapur 413001, Maharashtra
Mobile 9890918687
Email : solapur@niralibooks.com

marketing@pragationline.com | www.pragationline.com

Also find us on www.facebook.com/niralibooks

PREFACE

It gives me great pleasure to present the book **"Advanced Digital Signal Processing" (Elective III)** for the students of **Semester VI Third Year (T.Y.) B. Tech. Course Electronics and Telecommunication Engineering of Dr. Babasaheb Ambedkar Technological University (DBATU), Lonere, Dist. Raigad (Maharashtra).** This book is strictly as per the new revised syllabus 2020 Pattern, effective from the Academic Year July 2022-23. This book is an outcome of various lecture notes used for teaching the engineering students.

In New Revised Syllabus, there will Class Assessment (CA) 20 Marks, Mid Semester Examination (MSE) 20 Marks on First and Second units and End Semester Examination (ESE) 60 Marks. End Semester Examination will be based on all Five units.

The Theory Course will have 4 Credits.

The basic objective of this book is to bridge the gap between the vast contents of the reference books, written by the renowned International Authors and the concise requirements of Undergraduate Students. This book has been written in a comprehensive manner using Simple and Lucid language, keeping in mind students' requirements. The main emphasis has been given on exploring the basic concepts rather than merely the Information. Solved Examples and Exercises have been provided throughout the book and at the end of the Unit. Also, we have given **Model Question Papers** for practice at the end of book.

My special thanks to my family members, students and all those who directly or indirectly supported me in this project.

I also take this opportunity to express my sincere thanks to Shri. Dineshbhai Furia, Shri. Jignesh Furia, Mrs. Nirali Verma and entire team of Nirali Prakashan, namely Mrs. Deepali Lachake (Co-ordinator), who really have taken keen interest and untiring efforts in publishing this text.

I wish to make a special mention of the valuable contribution made by Late Shri M.P. Munde during the last two decades in reaching out to students, parents and teachers which eventually made **Nirali Prakashan, a brand of trust and quality in technical books.** Without his perseverance and zeal, successive editions would not have been possible.

The advice and suggestions of my esteemed readers to improve the text are most welcomed, and will be highly appreciated.

Pune **Author**

SYLLABUS

Unit I : Fundamentals of Multirate Systems **(7 Hours)**

Introduction, Basic multirate operations, Interconnection of building blocks, Polyphase representation, Mulstage implementation, Some application of multirate systems, Special filter and filter banks.

Unit II : Maximally Decimaled Filter Banks **(7 Hours)**

Introduction, Errors created in the QMF bank, A simple alias free QMF system, Power symmetric QMF banks, M-channel filter banks, Polyphase representation, Perfect reconstruction system, Alias free filter banks, Treestructured filter banks, Transmultiplexer.

Unit III : Paranitary Perfect Reconstruction Filter Banks **(7 Hours)**

Introduction, Lossless transfer matrices, Filter banks properties induced by paraunitariness, Two channel FIR paraunitary QMF banks, Two channel para unitary QMF lattice, M - channel FIR paraunitary filter banks, Transform coding and LOT.

Unit IV : Linear Phase and Cosine Modulated Filter Banks **(7 Hours)**

Introduction, Some necessary conditions, Lattice structure for linear phase FIR PR banks, formal synthesis of linear phase FIRPR QMF Lattice. Pseudo QMF banks, Design of the Pseudo QMF bank, Efficient poly phase structure, Cosine modulated perfect reconstruction system.

Unit V : The Wavelet Transform and its Relation to Multirate Filter Banks **(7 Hours)**

Introduction, Background and outline, Short time fourier transform, The Wavelet transform, DT orthonormal Wavelets, Continuous time orthonormal Wavelet basis.

Multidimensional, Multivariable and Lossless Systems : Introduction, Multidimensional signals, Sampling a multidimensional Signals, Multirate fundamentals. Review of discrete time multi-input multi-output LTI System, Para-Unitary and lossless system.

CONTENTS

Unit I : Fundamental of Multi-Rate Systems **1.1-1.24**

1.1 Concept of Multirate DSP 1.1
1.2 Introduction 1.2
1.3 Basic multi-rate operations 1.2
1.3.1 Decimation 1.3
1.3.2 Interpolation 1.4
1.3.3 Frequency Transforms of Decimated and Expanded Signals 1.5
1.3.4 Frequency Transform of Decimated Signal 1.5
1.3.5 Frequency Transform of an Expanded Signal 1.5
1.3.6 Sampling Rate Conversion 1.6
1.4 Interconnection of Building Blocks 1.7
1.5 Polyphase Representation 1.7
1.5.1 Polyphase Representation of Decimation Filter 1.9
1.5.2 Polyphase Type 2 Representation of Interpolation Filter 1.9
1.6 Multistage Implementation 1.9
1.7 Applications of Multi-rate Processing 1.10
1.7.1 Multi-rate Signal Processing in Music 1.10
1.7.2 Multi-rate Signal Processing in Radar 1.11
1.7.3 Multi-rate Signal Processing in Communication 1.11
1.7.4 Multi-rate Signal Processing in Image 1.11
1.7.5 Sub-Band Coding 1.11
1.8 Special FIlter and FIlter Banks 1.13
1.8.1 M-th Band Filters or Nyquist (M) Filters 1.13
1.8.2 Half-Band Filters 1.15
1.8.3 Zero-phase FIR Half-hand Filters 1.16
1.8.4 Design of Zero-phase Half-band and M-th Band Filters 1.16
1.8.5 Eigen Filter Designs 1.16
1.8.6 Optimal Lowpass Eigenfilters 1.16
1.8.7 Design of Mth Band Filter Using Eigenfilter Approach 1.18
1.8.8 Complimentary Filters 1.19
1.8.9 Relation Between Nyquist (M) Filter and Power Complementary Filters 1.20
1.8.10 Applications of Special Filters 1.20
- **Exercise** 1.24

Unit II : Maximally Decimated Filter Banks **2.1-2.12**

2.1 Introduction 2.1
2.2 Errors Created in The QME bank 2.2
2.2.1 Aliasing and Imaging 2.2
2.2.2 Aliasing Cancellation Using Synthesis Filters Design 2.3
2.2.3 Amplitude and Phase Distortions 2.5

2.3 A Simple Alias-Free Qmf System 2.5
2.3.1 Polyphase Representation of Alias-Free QMF System 2.6
2.4 Power Symmetric QMF Banks 2.7
2.4.1 Power Symmetry Property 2.8
2.4.2 Poles of Power Symmetric Elliptic Filters 2.8
2.5 M-channel Filter Banks 2.9
2.6 Polyphase Representation 2.10
2.7 Perfect Reconstruciton system 2.11
• **Exercise** 2.12

Unit III : Paraunitary Perfect Reconstruction (PR) Filter Banks 3.1-3.14

3.1 Introduction 3.1
3.2 Lossless Transfer Matrices 3.1
3.2.1 Definition and Properties of Paraunitary System 3.1
3.2.2 Operation Preserving Para-unitary and Lossless Properties 3.2
3.2.3 Para-unitary Filter Banks 3.5
3.3 Filter Bank Properties Induced by Paraunitariness 3.5
3.3.1 Relation between Analysis and Synthesis Filters 3.6
3.3.2 Other Properties Induced by Para-unitary E(z) = Poly-phase Matrix for the Analysis Filters 3.6
3.4 Two Channel fir para-unitary QMF banks 3.7
3.4.1 Properties of Para-unitary E(z) 3.8
3.4.2 Design of PR QMF Bank 3.8
3.4.3 Properties of Two Channel FIR PR QMF Bank 3.9
3.5 Two Channel Para-unitary QMF Lattice 3.9
3.5.1 Properties of Two Channel Para-unitary QMF Lattice 3.10
3.5.2 Advantages of QMF Lattice 3.11
3.6 M-channel FIR para-unitary Filter Banks 3.11
3.6.1 Design of M = 3 FIR Filter Banks 3.11
3.7 Transfrom Coding and Lot 3.13
3.7.1 Transform Coding 3.13
3.7.2 LOT 3.13
• **Exercise** 3.14

Unit IV : Linear Phase and Cosine Modulated Filter Banks 4.1-4.22

4.1 Introduction 4.1
4.2 Some Necessary Conditions 4.1
4.3 Lattice Structures for Linear Phase FIR PR Banks 4.1
4.3.1 Design of an FIR Linear Phase PR QMF Bank 4.2
4.3.2 Design of General Linear Phase FIR PR QMF Bank 4.3
4.3.3 Design of Linear Phase FIR PR Lattice for J = 31 4.4
4.4 Formal Synthesis of Linear Phase FIR PR QMF Lattice 4.6
4.4.1 Condition on $H_0(z)$ and $H_1(z)$ for Lattice Realization 4.6
4.5 Cosine Modulated Filter Banks 4.7
4.6 Pseudo QMF Banks 4.7
4.6.1 Generation of M Real Coefficient Analysis Filters 4.7
4.6.2 Definition of the Real Coefficient Analysis Filters 4.9
4.6.3 Alias Cancelation 4.10

4.6.4 Eliminating Phase Distortion 4.10
4.6.5 Closed Form Expression for Filters 4.10
4.7 Design of the Pseudo QMF bank 4.11
4.7.1 The Design of the Pseudo QMF Bank 4.11
4.8 Efficient Polyphase Structure 4.12
4.9 Cosine modulated perfect reconstruciton system 4.14
4.9.1 DCT and DST Matrices for Cosine Modulation 4.14
4.9.2 Expression for Polyphase Matrix E(z) 4.15
4.9.3 Design Procedure 4.16
4.10 Complexity Comparison 4.19
4.10.1 Design Complexity 4.19
4.10.2 Implementation Complexity 4.20
4.11 Implementing Cosine Modulation with DCT and DST 4.20
4.12 Advantages of The Cosine Modulated System 4.21
- **Exercise** 4.22

Unit V : The Wavelet Transform and Its Relation to Multirate Filter Banks 5.1-5.46

5.1 The Wavelet Transform and its Relation to Multirate Filter Banks 5.1
5.1.1 Introduction 5.1
5.1.2 Bankground and Outline 5.1
5.1.3 The Short-Time Fourier Transform 5.2
5.1.4 The Wavelet Transform 5.2
5.1.5 Time-Frequency Representations 5.3
5.1.6 Outline 5.4
5.1.7 The Short Term Fourier Transform (STFT) 5.4
5.1.8 Interpretation of STFT using Bandpass Filters 5.5
5.1.9 Traditional Fourier Transform as a Bank of Filters 5.5
5.1.10 The STFT as a Bank of Filters 5.6
5.1.11 The Uniform DFT Filter Bank 5.7
5.1.12 Choice of V(n) and Time-frequency Tradeoff 5.8
5.1.13 Time-Frequency Representation and Decimation 5.8
5.1.14 The Time-Frequency Grid 5.9
5.1.15 Inversion of the STFT 5.9
5.1.16 Filter Bank Interpretation of the Inverse Transform 5.9
5.1.17 Discrete-time STFT 5.9
5.1.18 Continuous – Time Case 5.10
5.2 The Wavelet Transfrom 5.10
5.2.1 Passing from STFT to Wavelets 5.11
5.2.2 Time-Frequency Grid 5.12
5.2.3 General Definition of Wavelet Transform 5.12
5.2.4 Inversion of Wavelet Transform 5.12
5.2.5 Filter Bank Interpretation of Inversion 5.13
5.2.6 Main Points of Wavelet Transform 5.13
5.2.7 Discrete-Time Wavelet Transform 5.14
5.2.8 Defining Discrete – Time Wavelet Transform 5.17
5.2.9 The Discrete-time Inverse Wavelet Transform 5.17

5.2.10 Summary 5.18
5.2.11 The Disk-Partition Diagram for Wavelet and STFT 5.19
5.3 Discrete-Time Orthogonal Wavelets 5.19
5.3.1 Two Channel Paraunitary QMF Banks 5.20
5.3.2 Orthonormality of Wavelet Basis 5.20
5.3.3 Paraunitariness of R(z) and Wavelet Orthonormality 5.21
5.3.4 Design of FIR PR QMF Bank Filter 5.21
5.3.5 Wavelet Orthonormality Theorem 5.21
5.4 Continuous-Time Orthonormal Wavelet Basis 5.21
5.5 Multidimensional, Multivariable and Lossless Systems 5.23
5.5.1 Introduction to Multidimensional Multirate System 5.23
5.5.2 Multidimensional Signals 5.24
5.5.3 The Fourier Transform of MD Signals 5.24
5.5.4 The Z-transform of MD Signals 5.25
5.5.5 Multidimensional Digital Filters 5.26
5.6 Sampling A Multidimentional Signal 5.29
5.6.1 Multidimensional Sampling 5.31
5.6.2 Sampling Density and Determinant of V 5.32
5.7 Multirate FundamentaLs for MD Signals 5.33
5.7.1 Basic Building Blocks and Tools 5.33
5.7.2 Polyphase Decomposition 5.36
5.7.3 Multidimensional Decimator (Transform Domain Analysis) 5.38
5.7.4 Multidimensional Expander(Transform) Domain Analysis 5.39
5.7.5 Polyphase Decomposition (Transform Domain Expression) 5.41
5.8 Review of Discrete Time Multi Input-Output LTI Systems 5.42
5.8.1 Multi-Input Multi-Output Systems 5.42
5.9 Paraunitary and Lossless Systems 5.45
5.9.1 Definition and Properties 5.45
- **Exercise** 5.46

- **Appendix A** **A.1 – A.14**
- **Appendix B** **B.1 – B.4**
- **Model Question Papers**
  - ➢ **Mid-Sem. Exam. (20 Marks)** **P.1 – P.1**
  - ➢ **End-Sem. Exam. (60 Marks)** **P.2 – P.2**

❋ ❋ ❋

UNIT - I

FUNDAMENTAL OF MULTI-RATE SYSTEMS

1.1 CONCEPT OF MULTIRATE DSP

Multirate DSP

- In all previous chapters, we have discussed time domain and frequency domain digital signal processing based on uniform and single sampling rate. However, in modern digital era, Multirate DSP is applied very efficiently in various applications such as Communication, Music, RADAR signals and Image Processing.
- Multirate means multiple sampling rates. Thus the processing of a digital signal x(n), at different sampling rates in different part of a system is called multirate DSP. We can achieve multirate DSP either in analog domain or in digital domain. The block diagram of analog domain multirate DSP is given in Fig. 1.1.

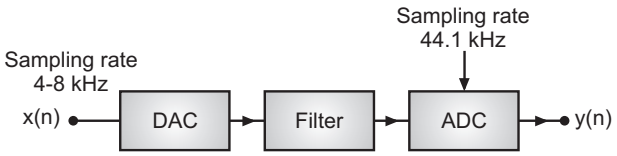


Fig. 1.1 : Analog domain multirate DSP

- Let us consider, we have music signal x(n), recorded at 48 KHz sampling rate in a studio. For recoding the same x(n) in CD, we need to decrease the 48 KHz sampling rate to 44.1 KHz. We can decrease the sampling rate by keeping 44.1 KHz sampling frequency of the ADC of Fig. 1.1.
- The main advantage of Analog Domain DSP is the selection of any sampling rate without bothering about its relation with old sampling rate. In this technique, we convert digital signal x(n) in analog using DAC, thus we introduces distortion in signal reconstruction. ADC also introduces quantization error.
- These two errors are eliminated in Digital Domain multirate Sampling. Multirate sampling is achieved by either Interpolation (Up Sampler) or Decimation (Down Sampler) process. Decimation is the process of reducing the sampling rate. Similarly, the interpolation is the process of increasing the sampling rate. The process of decimation is represented as $\downarrow D$ and the process of interpolation is denoted as $\uparrow I$.

Examples of Multirate DSP :

(i) In digital transmission systems like teletype and facsimile, low sampling rate is used for speech and different sampling rates are used for data transmission.

(ii) In digital audio system, different sampling rates for different applications are used. The details of multirate DSP for audio signals are given below:

Sr. No.	Description of Audio Applications	Sampling Rate Used
1.	Broadcasting	32 kHz
2.	Compact disc	44.1 kHz
3.	DVD	48 kHz
4.	Telephone	8 kHz

(iii) The details of multirate DSP for video signals are given below :

Sr. No.	Description of Video Applications	Sampling Rate Used
1.	Composite video signal for PAL	17.734475 MHz
2.	Composite video signal for NTSC	14.3181818 MHz
3.	Luminance video signal	13.5 MHz
4.	Color difference signal	6.75 MHz

Advantages and Disadvantages of Multirate DSP

The various advantages of multirate DSP are as given below:

- Computational requirements are less.
- Storage for filter coefficients are less.
- Finite word length effects are less.
- Filter order required in multirate application is low.
- Sensitivity to filter coefficient lengths is less.

There are two main disadvantages of multirate DSP:

- Severe problem of aliasing in decimation.
- Problem of pseudo images in interpolation.

Why Multirate Processing is Needed? Explain with Example

Decimation and interpolation are two primary operations in Multirate Processing to change the data rate. In decimation, sampling rate is reduced to compress the data and retaining only the desired information. On the other hand, Interpolation increases the sampling rate. For this reasons, we change the sampling rate :

(i) To process data easily and efficiently

(ii) To achieve compatibility with another digital system.

If we reduce the sampling rate of a signal from 100 kHz to 10 kHz without (using desired information) then we have reduced the computational burden in subsequent signal processing by a factor of 10.

Example of Decimation:

High quality music is recorded in a studio at a sampling rate of 48 kHz. However, if we want to play the music recorded in studio on CD (Compact Disc), then the Studio recorded music data must first be decreased to 44.1 kHz using Decimation.

Example of Interpolation:

To avoid the problems in analog filters, Interpolation is employed in the CD player. This is achieved by increasing, the sampling frequency of the data by 4 to 176.4 KHz before it is applied to the DAC.

Reasons for using Multirate DSP for Filter Design

- Requires reduced computation.
- Requires less storage space.
- Filter design is simple.
- Finite word length effects are less.

The demerits of the systems are that proper control structure is required in implementing the system and proper value of L and D should be chosen.

1.2 INTRODUCTION

Since the early 1980s, multi-rate systems are commonly used for audio and video processing, communication systems and transform analysis. We improve the performance and computational efficiency by using multi-rate systems.

1.3 BASIC MULTI-RATE OPERATIONS

There are two basic multi-rate operations i.e. decimation and interpolation. In decimation operation, we decrease the sampling rate of a discrete signal.

However, in interpolation operation, we increase the sampling rate of a discrete signal. Multi-rate systems are also used for sampling-rate conversion. In sampling-rate conversion, we use both interpolation and decimation operations.

1.3.1 Decimation

- In decimation, we decrease the sampling rate of a discrete signal x(n). The M-fold decimation of a discrete signal x(x) is given by Equation 1.1. The sampling rate is reduced from Fs to Fs/M, by discarding M-1 samples for every M samples in the original discrete signal x(n).

$$y(n) = V[n\,M] = \sum_{k=-\infty}^{\infty} h(k)\, x\,(nM - K) \qquad \ldots (1.1)$$

Here, $y(n)$ = Decimated discrete signal

$V(n)$ = Output of digital anti-aliasing low pass filter

$h(k)$ = Impulse response of filter

The block diagram of decimation process is given in Fig. 1.2.

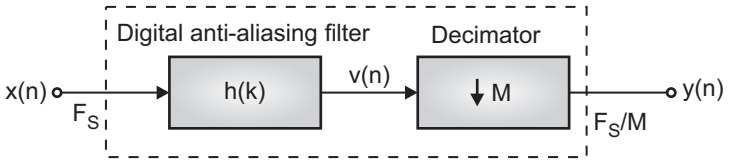


Fig. 1.2 : Block diagram of decimation

- The digital anti-aliasing LPF prevents the occurring of aliasing due to lower sampling rate. We illustrate the concept of 3-fold decimation i.e. M = 3 in Fig. 1.3.

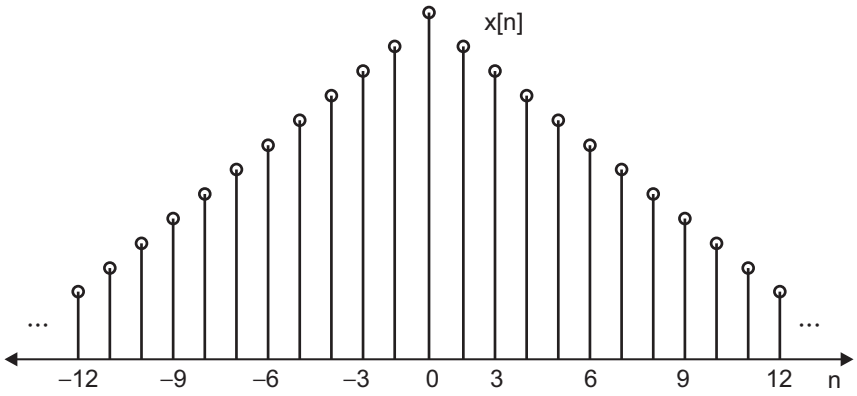
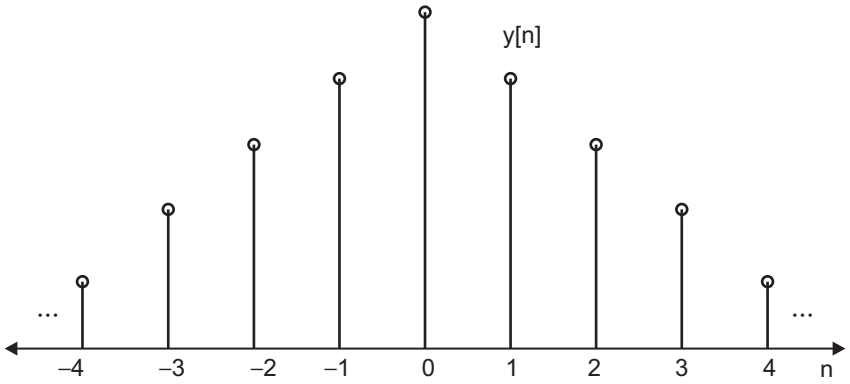


Fig. 1.3(a)



(b)

Fig. 1.3(b) : Decimation of x(n) by factor of 3

- From Fig. 1.3(a). It is observed that the samples of x(n) corresponding to n = 1, 2, 4, 5, 7, 8, 10 and 11 are lost in right hand side due to decimation process.

- Similarly, the samples of x(n) corresponding to n = –1, –2, – 4, –5, –7, – 8, –10 and – 11 are lost in left hand side due to decimation process. The decimated signal y(n) is shown in Fig. 1.2(b). The decimated signal y(n) is slower than the original signal x(n) by a factor of M = 3 of the sampling frequency of x(n) is Fs, then the sampling frequency of y(n) is $\frac{F_s}{M}$.

1.3.2 Interpolation

Interpolation is an information preserving operation. In this process, all samples of original discrete signal x(n) are kept as it is, in the expanded signal y(n). The L fold interpolation of a discrete signal x(n) is given as:

$$y(n) = L \sum_{k=-\infty}^{\infty} h(k)\, W(n-k) \quad \ldots (1.2)$$

Where,
$$w(n) = \begin{cases} x(n/L) & \text{for } \frac{n}{L} \text{ is integer} \\ 0 & \text{for } \frac{n}{L} \text{ is non-integer} \end{cases}$$

- The block diagram of interpolation process is depicted in Fig. 1.3. In this process, we insert L-1 zero-valued samples between two consecutive samples. The sampling rate is increased from Fs to LFs. The interpolation process is followed by a unique digital low-pass filter called anti-imaging filter.

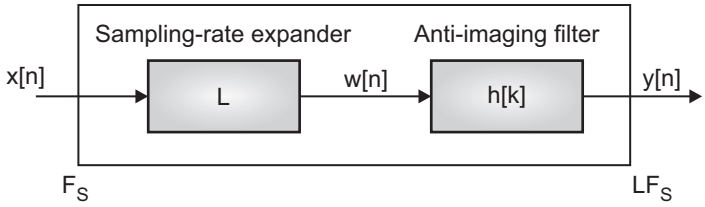


Fig. 1.4 : Block diagram notation of interpolation, by a factor of L

- Although the expansion process does not cause any aliasing in the interpolated signal, it does however yield undesirable replica in the frequency spectrum.
- Fig. 1.5 shows the 3-fold interpolation of the discrete signal x(n) for L=3. The insertion of zero effectively attenuates the signal by L, so the output of the anti-imaging filter must be multiplied by L to maintain the same signal amplitude.

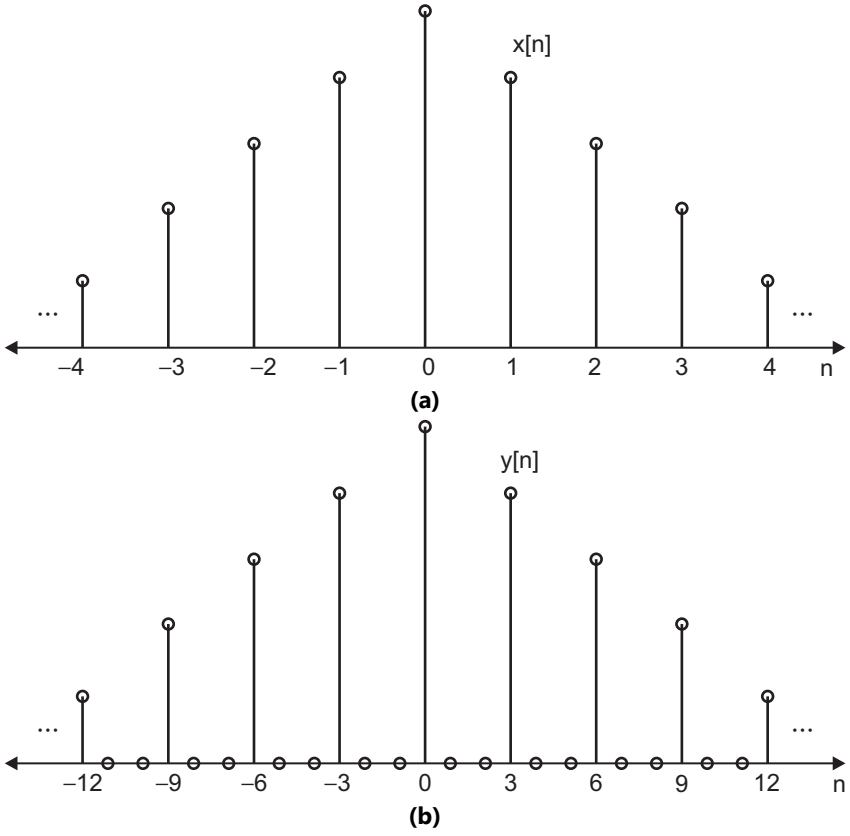


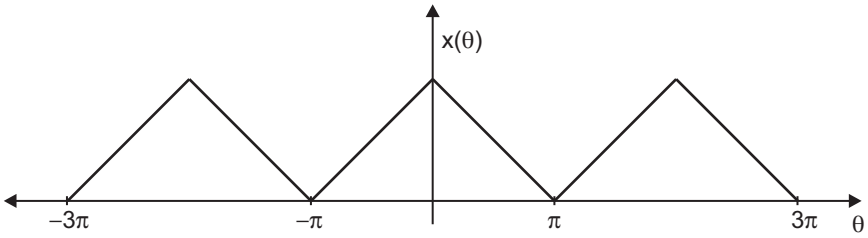
Fig. 1.5 : Interpolation of a discrete-time signal by a factor of 3

1.3.3 Frequency Transforms of Decimated and Expanded Signals

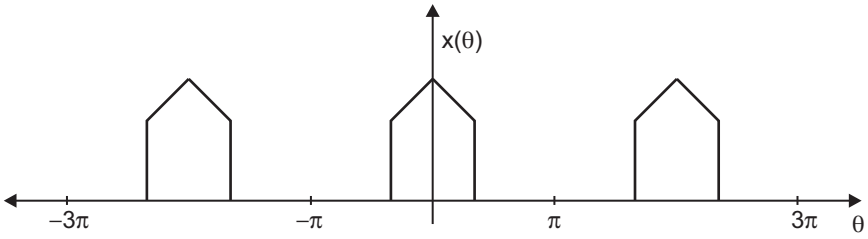
The analysis of decimation and interpolation is understood by seeing the frequency spectra. The frequency spectra are calculated using FFT (Fast Fourier Transform).

1.3.4 Frequency Transform of Decimated Signal

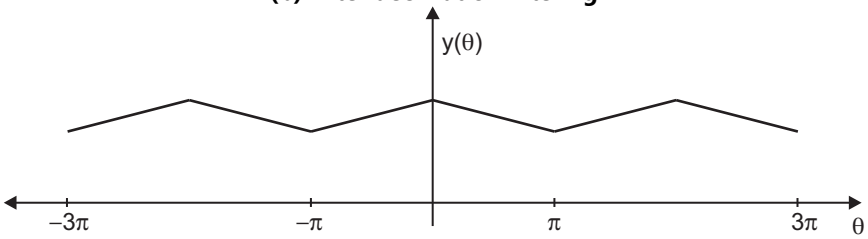
The implications of aliasing caused by decimation are similar to the case of sampling an analog signal the aliasing phenomenon of decimated signal for M = 3 is shown in Fig. 1.6.



(a) Fourier transform of the original signal



(b) After decimation filtering

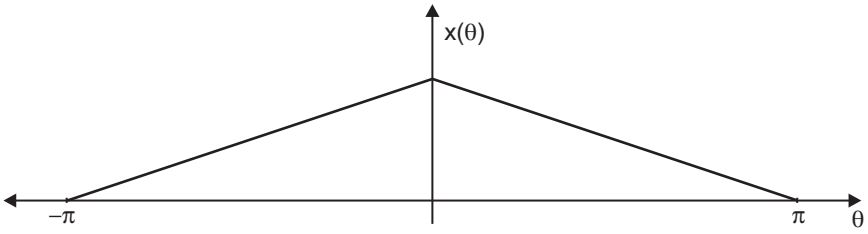


(c) Fourier transform of the decimated signal

Fig. 1.6 : Aliasing caused by decimations

1.3.5 Frequency Transform of an Expanded Signal

The effect of expansion on a signal in frequency domain is shown in Fig. 1.7.



(a)

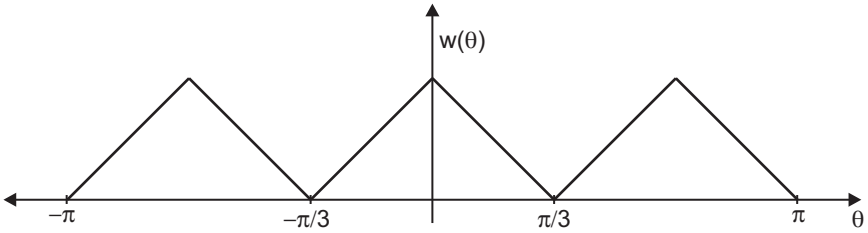


Fig. 1.7(b)