Mathematics

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INTRODUCTION

THE history of the beginnings of mathematics in the sense in which we understand the term is the history of mathematics in Greece; for it was the Greeks who first conceived the notion of mathematics as a science in and for itself, and it was they who established mathematics as a logical system based upon a few elementary principles, principles which they were the first to lay down and which remain substantially unshaken to the present day.

If it is true that any one who desires to get to the root of a subject should study it historically, it should seem almost superfluous to urge the student of mathematics to go to the sources. But it is not only the mathematician who should study the original works of the Greek masters. Some acquaintance with them is necessary to any one who would understand the Greek genius in all its many-sided aspects and in some sort feel the inspiration which all succeeding ages

have drawn from the imperishable monuments which it has left us. For the mathematics of the Greeks was on a level with their pioneer work in all the other intellectual pursuits, literature, philosophy, art and science, which make life worth living to a rational human being. In their mathematics we see the same untrammelled intellectual freedom, the same truthfulness and directness, that characterize their work in other fields, and even the same love of beauty. Aristotle, for instance, expressly combats the view that mathematics has nothing to do with beauty and the good; on the contrary, he says, what could be more beautiful than the order, symmetry and definiteness which mathematics exhibits in a very special degree?

The use of mathematics, in the view of the great Greek teachers, was to train the mind and to enable the soul to get a glimpse of truth; its practical application to everyday problems was of no account in comparison. In the *Republic* of Plato Socrates rallies Glaucon for thinking well of astronomy because it enables us to observe seasons, months and years. "You amuse me," he says,

"by your evident alarm that the multitude should think that you insist upon useless studies. Yet indeed it *is* no easy matter, but on the contrary a very difficult one, to believe that in the midst of these studies an organ of our souls is being purged from the blindness, and quickened from the deadness, occasioned by other pursuits, — an organ the preservation of which is of more importance than a thousand eyes; because only by it can truth be seen. Consequently those who think with us will bestow unqualified approbation on the studies you prescribe, while those who have no inkling at all of this doctrine, will think them valueless, because they see no considerable advantage to be gained from them beyond their practical applications."

Archimedes, again, whose clever mechanical inventions, like the screw for pumping water and the engines of war which kept the Romans so long at bay when besieging Syracuse, had (as Plutarch relates) brought him the reputation of more than human wisdom, thought nothing of such things; nay, he regarded as ignoble and sordid the business of mechanics and every sort of art that is

directed to use and profit, and placed his whole ambition in these speculations, the beauty and subtlety of which is untainted by any admixture of the common needs of life. In this spirit it was that he ordered to be engraved on his tomb a representation of a cylinder and a sphere inscribed in it, with the ratio (3 : 2) of their surfaces and volumes which he had discovered.

And may we not claim that today, in the welter of things resulting from the Great War, it should do every one good to turn aside and breathe for a while the pure atmosphere of abstract thought which we find in Greek mathematics, that "independent world," as Wordsworth eloquently called it, "created out of pure intelligence"?

Indeed there could not be a more appropriate time for the appearance of a Series such as that of which the present volume forms a part.

THOMAS LITTLE HEATH

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MATHEMATICS

MATHEMATICS

I. PRELIMINARY SURVEY

1. Leading Contributions of Greece

General Debt to Greece. If a man of scholarly tastes, even though gifted with only moderate intelligence, were asked where one should seek the ancient home of philosophy, he would wonder at the motive that could prompt a question upon which the opinion of the world is so unanimous. If the architect were asked a similar question with respect to his chosen field of labor, he too would marvel how a man of even modest attainments could have cause for doubt as to the answer. And so we might run the gamut of intellectual pursuits; of the graphic and plastic arts; of dramatic, musical, and forensic expression; of politics, and of all that makes for the higher life, and the answer would be the same, — the culmination of each in ancient times is found in the Greek civilization.

Precisely wherein lay the secrets of this power of Greece? Sir Thomas Heath, than whom no one is more conversant with her mathematical ideals, in an address upon *Greek Mathematics and Science*, has thus summarized the special aptitudes which her scholars possessed for success in these fields:

"They had, first, a love of knowledge for its own sake amounting, as Butcher says, to an instinct and a passion; secondly, a love of truth and a determination to see things as they are; thirdly, a remarkable capacity for accurate observation. Fourthly, while eagerly assimilating information from all quarters, from Egypt and Babylon in particular, they had an unerring instinct for taking what was worth having and rejecting the rest. As one writer has said, 'It remains their everlasting glory that they discovered and made use of the serious scientific elements in the confused and complex mass of exact observations and superstitious ideas which constituted the priestly wisdom of the East and threw all the fantastic rubbish on one side.' Fifthly, they possessed a speculative genius unrivalled in the world's history.

"It was this unique combination of gifts which

qualified the Greeks to lead the world in all the intellectual pursuits that make life worth living.

"Last, but not less important, the Greeks possessed the advantage over the Egyptians and Babylonians of having no priesthood which could monopolize learning as a preserve of its own, with the inevitable result of sterilizing it by keeping it bound up with religious dogmas and prescribed and narrow routine."

Special Debt of Mathematics. In view of these aptitudes of the Greeks, it is not to be wondered that the mathematician, — whose thought is so closely allied to the speculations of the philosopher, and whose domain of interest once included music and astronomy, and who not infrequently offered a helping hand to all the various fine arts, — should likewise claim Greece for the birthplace of his own chosen science.

Whether the peculiar traits of the Hellenic mind already mentioned were sufficient to account for the most logical number theory, for example, that was known in ancient times, while Greece in the same period was developing the most graceful dance of all the ages, is a further question for the philosopher; but the fact is patent, and it is now

proposed to accept that fact for the time being, and to consider some of the larger features of Greek mathematics, discussing later the details which they included.

Mathematics is a term that is loosely used at present to cover various subjects which, except by tradition, are not always closely related. We speak of arithmetic as one of these subjects, but the term has a different meaning from that ascribed to it by the classical writers, and its present connection with demonstrative geometry, for example, is nearly negligible. Music has not been considered a part of mathematics for some four centuries, although for at least two thousand years it was so looked upon in all of Western Europe. The name of algebra is comparatively modern, and the subject as today conceived was almost unknown to the Greeks. Hence it might seem that, after all, the roots of the science as we understand it must be sought elsewhere than in Athens, Alexandria, and the Aegean Islands. This, however, is not the case, and we shall presently see that each of the several branches sprang from the sturdy trunk of Greek *mathesis*.

Pre-Grecian Mathematics. It is quite evident, however, that mathematics existed before the Grecian civilization was born, and that the Greeks derived much knowledge of the subject from Egypt and Babylon, possibly from Crete, and possibly also from India and China, — but none of this was what we know as the distinctively Greek mathematics, and little of it represents the mathematical disciplines as we now recognize them. Before the seventh century B.C., which is the period of the birth of the science in Ionia, Egypt had contributed a rather crude system of numerals, no worse and but little better than various others of the early races. She had done something with unit fractions (fractions like $\frac{1}{4}$, with the numerator unity) and had also contributed a few number puzzles which we would now solve by algebra, but which her own scholars solved by trial. Her engineers had shown great skill in running levels and in establishing planes with a given gradient, and her *harpedonaptae* ("rope-stretchers," surveyors) had developed certain convenient rules in their useful craft.

The scientists of Mesopotamia, chiefly

astrologers, no doubt, had developed a cumbersome system of numerals and had prepared elaborate sets of tables of products, powers, quotients, and roots; and they, like the Egyptians, had made a slight advance in the study of fractions. Like the Egyptians also, they had developed a few simple rules of mensuration, and had made a few applications of their mathematics to a primitive type of astronomy.

The history of ancient India is almost completely sealed to us. We have the Mahabhārata, the Rāmayāna, and the Vedic literature; but from none of these sources do we obtain any information of value relating to the early mathematics of her people. China offers a better field for study, but all that her pre-Grecian period has to offer is possibly (for the texts may have been considerably corrupted) a few records of eclipses and a certain number of interesting rules and puzzles in mensuration.

In the eastern littoral region of the Mediterranean, where the trade from the Orient branched towards Egypt on the south and the distributing centers of Asia Minor on the north, there developed, so tradition says,

a fairly elaborate commercial arithmetic. This is quite reasonable of belief, since the same was true of the country watered by the Tigris and the Euphrates. Indeed, the early home of this humble type of mathematics may very likely have been located in Mesopotamia instead of Phœnicia.

So we find that, long before the warlike Dorians had established their dominion in Peloponnesus, computation, mensuration, and, no doubt, considerable number mysticism were known in various parts of the ancient world.

General Nature of the Greek Contribution. What, then, were the mathematical contributions peculiar to the Greeks? Did these people simply accept the accumulated wisdom of the past and add to it their more mature quota of knowledge, or were they able to contribute to the world something that was new, becoming creators instead of mere transmitters?

The answer is very simple and very direct: Within four centuries after Thales (*c.* 575 B.C.) founded the Ionian School of philosophy, there had developed in the Greek civilization an entirely new type of mathematics,

based upon new principles, and having new aims and new methods. It is in this epoch-making advance that all subsequent mathematics finds its beginning, and to this beginning the world is largely indebted for the basal principles upon which rests our present science of number, form, and space. In particular, we shall find that it was in these centuries that the foundations were laid for our modern theory of numbers, for our elementary geometry, for the study of conics and higher plane curves, for mathematical mechanics, and even, though more remotely, for the integral calculus. There have been numerous epoch-making periods in the history of mathematics, but considering all the circumstances and results, it is certain that there has never been a more significant one than that which included the names of Thales, Pythagoras, Eudoxus, Plato, Aristotle, Euclid, Apollonius, and Archimedes.

The three centuries that followed the death (*c.* 501 B.C.) of Pythagoras saw the foundations laid for geometry as a logical science, saw written the greatest textbook of the world, the *Elements* of Euclid, and saw mechanics placed upon a mathematical basis,

infinite and higher series added to the mathematical store; conic sections then reached its highest point in ancient times, algebra approached the scientific stage, higher plane curves were made a subject of investigation, and the best number system of the pre-Christian world was developed and brought into use; and all this was done through the intellectual prowess of Greek scholars.

This is, however, by no means the complete record of their achievements in the field of mathematics, and we shall presently find that to them are also due the first steps of great significance in trigonometry, in geodesy, and in the application of mathematics to such physical problems as their scientists had at that time developed.

2. Leading Contributions of Rome

Roman Ideals. It is a mere commonplace to say that the mind of Rome was radically different from the mind of Greece, and wherein that difference lay is set forth in the appropriate volumes in this series. Indebted to Greece for her initial inspiration, Rome also produced a noble and a notable