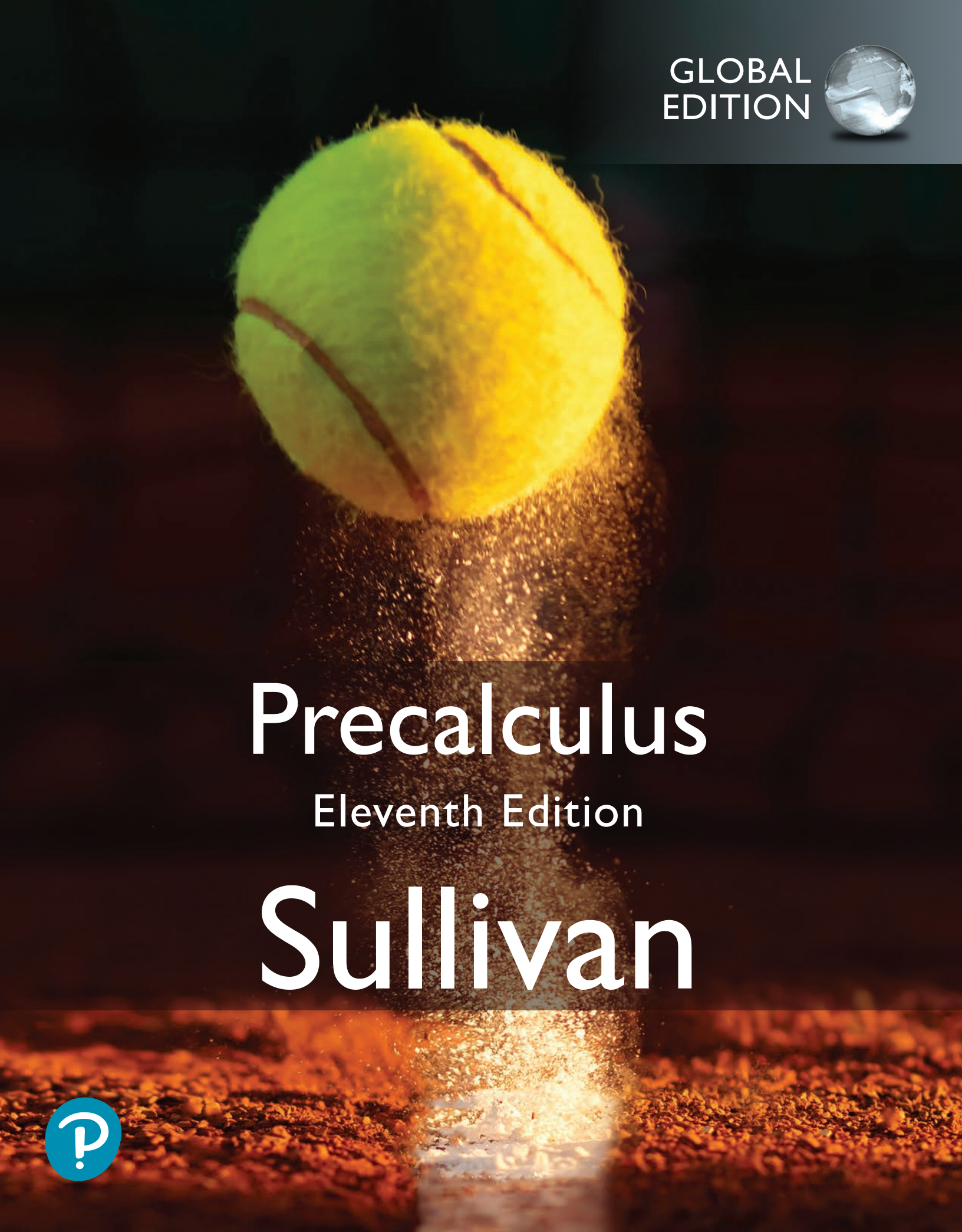
GLOBAL EDITION



Precalculus

Eleventh Edition

Sullivan

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry–your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this text, you will build the knowledge and skills you need to be successful. Here's how you can use the text to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't … the text has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

Use the Features

I use many different methods in the classroom to communicate. Those methods, when incorporated into the text, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it) to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this text. Refer to "Prepare for Class," "Practice," and "Review" at the front of the text. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your text.

Please do not hesitate to contact me through Pearson, with any questions, comments, or suggestions for improving this text. I look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan

This page is intentionally left blank

Prepare for Class "Read the Book"

Feature	Description	Benefit	Page
Every Chapter Opener begins with . . .			
Chapter-Opening Topic & Project	Each chapter begins with a discussion of a topic of current interest and ends with a related project.	The project lets you apply what you learned to solve a problem related to the topic.	294
Internet-based Project	The projects allow for the integration of spreadsheet technology that you will need to be a productive member of the workforce.	The projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest.	396
Every Section begins with . . .			
LEARNING OBJECTIVES 2	Each section begins with a list of objectives. Objectives also appear in the text where the objective is covered.	These focus your study by emphasizing what's most important and where to find it.	319
Sections contain . . .			
PREPARING FOR THIS SECTION	Most sections begin with a list of key concepts to review with page numbers.	Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward.	315
Now Work the **'Are You Prepared?' Problems**	Problems that assess whether you have the prerequisite knowledge for the upcoming section.	Not sure you need the Preparing for This Section review? Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it!	315, 326
Now Work PROBLEMS	These follow most examples and direct you to a related exercise.	We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read.	322, 327
WARNING	Warnings are provided in the text.	These point out common mistakes and help you to avoid them.	349
Exploration and **Seeing the Concept**	These graphing utility activities foreshadow a concept or solidify a concept just presented.	You will obtain a deeper and more intuitive understanding of theorems and definitions.	310, 335
In Words	These provide alternative descriptions of select definitions and theorems.	Does math ever look foreign to you? This feature translates math into plain English.	332
Calculus	These appear next to information essential for the study of calculus.	Pay attention–if you spend extra time now, you'll do better later!	90, 299, 322
SHOWCASE EXAMPLES	These examples provide "how-to" instruction by offering a guided, step-by-step approach to solving a problem.	With each step presented on the left and the mathematics displayed on the right, you can immediately see how each step is used.	261
Model It! Examples and Problems	These examples and problems require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple headings.	It is rare for a problem to come in the form "*Solve the following equation.*" Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models to find a solution to the problem.	339, 368
NEW! Need to Review?	These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note is back-referenced to the chapter, section and page where the concept was originally discussed.	Sometimes as you read, you encounter a word or concept you know you've seen before, but don't remember exactly what it means. This feature will point you to where you first learned the word or concept. A quick review now will help you see the connection to what you are learning for the first time and make remembering easier the next time.	308

Practice "Work the Problems"

Feature	Description	Benefit	Page
'Are You Prepared?' Problems	These assess your retention of the prerequisite material you'll need. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature.	Do you always remember what you've learned? Working these problems is the best way to find out. If you get one wrong, you'll know exactly what you need to review and where to review it!	332, 340
Concepts and Vocabulary	These short-answer questions, mainly Fill-in-the-Blank, Multiple-Choice and True/False items, assess your understanding of key definitions and concepts in the current section.	It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary.	326
Skill Building	Correlated with section examples, these problems provide straightforward practice.	It's important to dig in and develop your skills. These problems provide you with ample opportunity to do so.	326–328
Applications and Extensions	These problems allow you to apply your skills to real-world problems. They also allow you to extend concepts learned in the section.	You will see that the material learned within the section has many uses in everyday life.	329–331
NEW! Challenge Problems	These problems have been added in most sections and appear at the end of the Application and Extensions exercises. They are intended to be thought-provoking, requiring some ingenuity to solve.	Are you a student who likes being challenged? Then the Challenge Problems are for you! Your professor might also choose to assign a challenge problem as a group project. The ability to work with a team is a highly regarded skill in the working world.	331
Explaining Concepts: Discussion and Writing	"Discussion and Writing" problems are colored red. They support class discussion, verbalization of mathematical ideas, and writing and research projects.	To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging, but you'll get out what you put in.	331
Retain Your Knowledge	These problems allow you to practice content learned earlier in the course.	Remembering how to solve all the different kinds of problems that you encounter throughout the course is difficult. This practice helps you remember.	331
Now Work PROBLEMS	Many examples refer you to a related homework problem. These related problems are marked by a pencil and orange numbers.	If you get stuck while working problems, look for the closest Now Work problem, and refer to the related example to see if it helps.	324, 325
Review Exercises	Every chapter concludes with a comprehensive list of exercises to pratice. Use the list of objectives to determine the objective and examples that correspond to the problems.	Work these problems to ensure that you understand all the skills and concepts of the chapter. Think of it as a comprehensive review of the chapter.	391–394

Review "Study for Quizzes and Tests"

Feature	Description	Benefit	Page
The Chapter Review at the end of each chapter contains . . .			
Things to Know	A detailed list of important theorems, formulas, and definitions from the chapter.	Review these and you'll know the most important material in the chapter!	389–390
You Should Be Able to . . .	Contains a complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective.	Do the recommended exercises and you'll have mastered the key material. If you get something wrong, go back and work through the objective listed and try again.	390–391
Review Exercises	These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section.	Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place.	391–394
Chapter Test	About 15–20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes!	Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor's test. If you get a problem wrong, you can watch the Chapter Test Prep Video.	394
Cumulative Review	These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review. When you use them in conjunction with the Retain Your Knowledge problems, you will be ready for the final exam.	These problem sets are really important. Completing them will ensure that you are not forgetting anything as you go. This will go a long way toward keeping you primed for the final exam.	395
Chapter Projects	The Chapter Projects apply to what you've learned in the chapter. Additional projects are available on the Instructor's Resource Center (IRC).	The Chapter Projects give you an opportunity to use what you've learned in the chapter to the opening topic. If your instructor allows, these make excellent opportunities to work in a group, which is often the best way to learn math.	396
Internet-Based Projects	In selected chapters, a Web-based project is given.	These projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest by using the Internet to research and collect data.	396

To the Memory of
My Mother and Father

Precalculus

Eleventh Edition

Global Edition

Michael Sullivan
Chicago State University

Pearson

Product Management: *Gargi Banerjee and Neelakantan K.K.*

Content Strategy: *Shabnam Dohutia, Amrita Naskar, Deeptesh Sen*

Supplements: *Bedasree Das*

Digital Studio: *Vikram Medepalli and Abhilasha Watsa*

Rights and Permissions: *Anjali Singh and Ashish Vyas*

Cover Photo Credit: Raul Jichici/Shutterstock

Please contact https://support.pearson.com/getsupport/s/contactsupport with any queries on this content.

Pearson Education Limited

KAO Two

KAO Park

Harlow

CM17 9SR

United Kingdom

and Associated Companies throughout the world

Visit us on the World Wide Web at: www.pearsonglobaleditions.com

The rights of Michael Sullivan to be identified as the author of this work have been asserted by him in accordance with the Copyright, Designs and Patents Act 1988.

Authorized adaptation from the United States edition, entitled Precalculus, *11th edition, ISBN 9780135189405, by Michael Sullivan, published by Pearson Education © 2020.*

ISBN-10: 1-292-44452-5
ISBN-13: 978-1-292-44452-9
eBook ISBN-13: 978-1-292-44447-5

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

1 23

Typeset by Straive
eBook formatted by B2R Technologies Pvt. Ltd.

Contents

Three Distinct Series

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

Flagship Series, Eleventh Edition

The Flagship Series is the most traditional in approach yet modern in its treatment of precalculus mathematics. In each text, needed review material is included, and is referenced when it is used. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra, Algebra & Trigonometry, Trigonometry: A Unit Circle Approach, Precalculus.*

Enhanced with Graphing Utilities Series, Seventh Edition

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and encounter ideas usually studied in later courses. Many examples show solutions using algebra side-by-side with graphing techniques. Using technology, the approach to solving certain problems differs from the Flagship Series, while the emphasis on understanding concepts and building strong skills is maintained: *College Algebra, Algebra & Trigonometry, Precalculus*.

Concepts through Functions Series, Fourth Edition

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. The approach supports the Rule of Four, which states that functions can be represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry*.

The Flagship Series

College Algebra, Eleventh Edition

This text provides a contemporary approach to college algebra, with three chapters of review material preceding the chapters on functions. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for trigonometry, finite mathematics, and business calculus.

Algebra & Trigonometry, Eleventh Edition

This text contains all the material in *College Algebra*, but also develops the trigonometric functions using a right triangle approach and shows how it relates to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Precalculus, Eleventh Edition

This text contains one review chapter before covering the traditional precalculus topics of polynomial, rational, exponential, and logarithmic functions and their graphs. The trigonometric functions are introduced using a unit circle approach and showing how it relates to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane and in space, including the dot and cross products, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is provided, but is optional. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Trigonometry: a Unit Circle Approach, Eleventh Edition

This text, designed for stand-alone courses in trigonometry, develops the trigonometric functions using a unit circle approach and shows how it relates to the right triangle approach. Vectors in the plane and in space, including the dot and cross products, are presented. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Preface to the Instructor

As a professor of mathematics at an urban public university for 35 years, I understand the varied needs of precalculus students. Students range from being underprepared with little mathematical background and a fear of mathematics, to being highly prepared and motivated. For some, this is their final course in mathematics. For others, it is preparation for future mathematics courses. I have written this text with both groups in mind.

A tremendous benefit of authoring a successful series is the broad-based feedback I receive from instructors and students who have used previous editions. I am sincerely grateful for their support. Virtually every change to this edition is the result of their thoughtful comments and suggestions. I hope that I have been able to take their ideas and, building upon a successful foundation of the tenth edition, make this series an even better learning and teaching tool for students and instructors.

Features in the Eleventh Edition

A descriptive list of the many special features of *Precalculus* can be found in the front of this text. This list places the features in their proper context, as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review it and to discuss it with your students at the beginning of your course. My experience has been that when students use these features, they are more successful in the course.

- **Updated! Retain Your Knowledge Problems** These problems, which were new to the previous edition, are based on the article *"To Retain New Learning, Do the Math,"* published in the *Edurati Review.* In this article, Kevin Washburn suggests that "the more students are required to recall new content or skills, the better their memory will be." The Retain Your Knowledge problems were so well received that they have been expanded in this edition. Moreover, while the focus remains to help students maintain their skills, in most sections, problems were chosen that preview skills required to succeed in subsequent sections or in calculus. These are easily identified by the calculus icon ( ). All answers to Retain Your Knowledge problems are given in the back of the text and all are assignable in MyLab Math.
- **Guided Lecture Notes** Ideal for online, emporium/redesign courses, inverted classrooms, or traditional lecture classrooms. These lecture notes help students take thorough, organized, and understandable notes as they watch the Author in Action videos. They ask students to complete definitions, procedures, and examples based on the content of the videos and text. In addition, experience suggests that students learn by doing and understanding the why/how of the concept or property. Therefore, many sections will have an exploration activity to motivate student learning. These explorations introduce the topic and/or connect it to either a real-world application or a previous section. For example, when the vertical-line test is discussed in Section 2.2, after the theorem statement, the notes ask the students to explain why the vertical-line test works by using the definition of a function. This challenge helps students process the information at a higher level of understanding.
- **Illustrations** Many of the figures have captions to help connect the illustrations to the explanations in the body of the text.
- **Graphing Utility Screen Captures** In several instances we have added Desmos screen captures along with the TI-84 Plus C screen captures. These updated screen captures provide alternate ways of visualizing concepts and making connections between equations, data and graphs in full color.
- **Chapter Projects**, which apply the concepts of each chapter to a real-world situation, have been enhanced to give students an up-to-the-minute experience. Many of these projects are new, requiring the student to research information online in order to solve problems.
- **Exercise Sets** The exercises in the text have been reviewed and analyzed, some have been removed, and new ones have been added. All time-sensitive problems have been updated to the most recent information available. The problem sets remain classified according to purpose.

  The ***'Are You Prepared?'*** problems have been improved to better serve their purpose as a just-in-time review of concepts that the student will need to apply in the upcoming section.

  The ***Concepts and Vocabulary*** problems have been expanded to cover each objective of the section. These multiple-choice, fill-in-the-blank, and True/False exercises have been written to also serve as reading quizzes.

  Skill Building problems develop the student's computational skills with a large selection of exercises that are directly related to the objectives of the section. ***Mixed Practice*** problems offer a comprehensive assessment of skills that relate to more than one objective. Often these require skills learned earlier in the course.

  Applications and Extensions problems have been updated. Further, many new application-type exercises have been added, especially ones involving information and data drawn from sources the student will recognize, to improve relevance and timeliness.

  At the end of Applications and Extensions, we have a collection of one or more ***Challenge Problems***. These problems, as the title suggests, are intended to be thought-provoking, requiring some ingenuity to solve. They can be used for group work or to challenge students.

The ***Explaining Concepts: Discussion and Writing*** exercises provide opportunity for classroom discussion and group projects.

Updated! ***Retain Your Knowledge*** has been improved and expanded. The problems are based on material learned earlier in the course, especially calculus-related material. They serve to keep information that has already been learned "fresh" in the mind of the student.

NEW Need to Review? These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note includes a reference to the chapter, section and page where the concept was originally discussed.

Content Changes to the 11th edition

- ***Challenge Problems*** have been added in most sections at the end of the Application and Extensions exercises. Challenge Problems are intended to be thought-provoking problems that require some ingenuity to solve. They can be used to challenge students or for group work.
- **Need to Review?** These margin notes provide a just-in-time review for a concept needed now, but covered in an earlier section of the book. Each note is back-referenced to the chapter, section and page where the concept was originally discussed.
- Additional **Retain Your Knowledge** exercises, whose purpose is to keep learned material fresh in a student's mind, have been added to each section. Many of these new problems preview skills required for calculus or for concepts needed in subsequent sections.
- **Desmos screen captures** have been added throughout the text. This is done to recognize that graphing technology expands beyond graphing calculators.
- Examples and exercises throughout the text have been augmented to reflect a broader selection of STEM applications.
- Concepts and Vocabulary exercises have been expanded to cover each objective of a section.
- Skill building exercises have been expanded to assess a wider range of difficulty.
- Applied problems and those based on real data have been updated where appropriate.

Appendix A

- Section A.10 Objective 3 now includes rationalizing the numerator
  - NEW Example 6 Rationalizing Numerators
  - Problems 69-76 provide practice.
- Section A.10 Exercises now include more practice in simplifying radicals

Chapter 1

- NEW Section 1.2 Example 9 Testing an Equation for Symmetry
- Section 1.3 has been reorganized to treat the slope-intercept form of the equation of a line before finding an equation of a line using two points.

Chapter 2

- NEW Section 2.1 Objective 1 Describe a Relation
- NEW Section 2.2 Example 4 Expending Energy
- NEW Section 2.4 Example 4 Analyzing a Piecewise-defined Function
- NEW Example 1 Describing a Relation demonstrates using the Rule of Four to express a relation numerically, as a mapping, and graphically given a verbal description.

Chapter 3

- Section 3.3 introduces the concept of concavity for a quadratic function
- NEW Section 3.3 Example 3 Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts
- Section 3.3 Example 8 Analyzing the Motion of a Projectile (formerly in Section 3.4)
- NEW Section 3.4 Example 4 Fitting a Quadratic Function to Data

Chapter 4

- Section 4.1 has been revised and split into two sections:
  - 4.1 Polynomial Functions
  - 4.2 Graphing Polynomial Functions; Models
- NEW Section 4.2 Example 2 Graphing a Polynomial Function (a 4th degree polynomial function)

Chapter 5

- Section 5.2 now finds and verifies inverse functions analytically and graphically.

Chapter 6

- NEW Section 6.1 Example 6 Field Width of a Digital Lens Reflex Camera Lens
- Section 6.4 and 6.5 were reorganized for increased clarity.

Chapter 7

- Sections 7.1 and 7.2 were reorganized for increased clarity.

Chapter 9

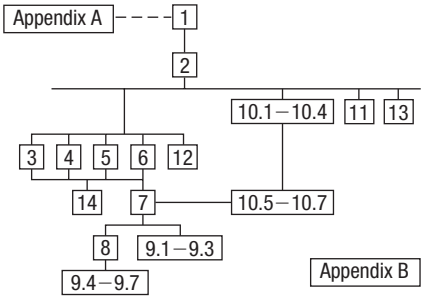
- Section 9.3 The complex plane; DeMoivre's Theorem, was rewritten to support the exponential form of a complex number.
  - Euler's Formula is introduced to express a complex number in exponential form.
  - The exponential form of a complex number is used to compute products and quotients.
  - DeMoivre's Theorem is expressed using the exponential form of a complex number.
  - The exponential form is used to find complex roots.

Chapter 11

- NEW Section 11.5 Example 1 Identifying Proper and Improper Rational Expressions

Using the Eleventh Edition Effectively with Your Syllabus

To meet the varied needs of diverse syllabi, this text contains more content than is likely to be covered in a *Precalculus* course. As the chart illustrates, this text has been organized with flexibility of use in mind. Within a given chapter, certain sections are optional (see the details that follow the figure below) and can be omitted without loss of continuity.



Appendix A Review

This chapter consists of review material. It may be used as the first part of the course or later as a just-in-time review when the content is required. Specific references to this chapter occur throughout the text to assist in the review process.

Chapter 1 Graphs

This chapter lays the foundation for functions.

Chapter 2 Functions and Their Graphs

Perhaps the most important chapter. Section 2.6 is optional.

Chapter 3 Linear and Quadratic Functions

Topic selection depends on your syllabus. Sections 3.2 and 3.4 may be omitted without loss of continuity.

Chapter 4 Polynomial and Rational Functions

Topic selection depends on your syllabus.

Chapter 5 Exponential and Logarithmic Functions

Sections 5.1–5.6 follow in sequence. Sections 5.7, 5.8, and 5.9 are optional.

Chapter 6 Trigonometric Functions

Section 6.6 may be omitted in a brief course.

Chapter 7 Analytic Trigonometry

Section 7.7 may be omitted in a brief course.

Chapter 8 Applications of Trigonometric Functions

Sections 8.4 and 8.5 may be omitted in a brief course.

Chapter 9 Polar Coordinates; Vectors

Sections 9.1–9.3 and Sections 9.4–9.7 are independent and may be covered separately.

Chapter 10 Analytic Geometry

Sections 10.1–10.4 follow in sequence. Sections 10.5, 10.6, and 10.7 are independent of each other, but each requires Sections 10.1–10.4.

Chapter 11 Systems of Equations and Inequalities

Sections 11.2–11.7 may be covered in any order, but each requires Section 11.1. Section 11.8 requires Section 11.7.

Chapter 12 Sequences; Induction; The Binomial Theorem

There are three independent parts: Sections 12.1–12.3; Section 12.4; and Section 12.5.

Chapter 13 Counting and Probability

The sections follow in sequence.

Chapter 14 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

If time permits, coverage of this chapter will give your students a beneficial head start in calculus.

Acknowledgments

Textbooks are written by authors, but evolve from an idea to final form through the efforts of many people. It was Don Dellen who first suggested this text and series to me. Don is remembered for his extensive contributions to publishing and mathematics.

Thanks are due to the following people for their assistance and encouragement to the preparation of this edition:

- From Pearson Education: Anne Kelly for her substantial contributions, ideas, and enthusiasm; Dawn Murrin, for her unmatched talent at getting the details right; Joseph Colella for always getting the reviews and pages to me on time; Peggy McMahon for directing the always difficult production process; Rose Kernan for handling liaison between the compositor and author; Peggy Lucas and Stacey Sveum for their genuine interest in marketing this text. Marcia Horton for her continued support and genuine interest; Paul Corey for his leadership and commitment to excellence; and the Pearson Sales team, for their continued confidence and personal support of Sullivan texts.
- Accuracy checkers: C. Brad Davis who read the entire manuscript and accuracy checked answers. His attention to detail is amazing; Timothy Britt, for creating the Solutions Manuals; and Kathleen Miranda and Pamela Trim for accuracy checking answers.

Finally, I offer my grateful thanks to the dedicated users and reviewers of my texts, whose collective insights form the backbone of each textbook revision.

James Africh, College of DuPage
Steve Agronsky, Cal Poly State University
Gererdo Aladro, Florida International University
Grant Alexander, Joliet Junior College
Dave Anderson, South Suburban College
Wes Anderson, Northwest Vista College
Richard Andrews, Florida A&M University
Joby Milo Anthony, University of Central Florida
James E. Arnold, University of Wisconsin-Milwaukee
Adel Arshaghi, Center for Educational Merit
Carolyn Autray, University of West Georgia
Agnes Azzolino, Middlesex County College
Wilson P. Banks, Illinois State University
Sudeshna Basu, Howard University
Timothy Bayer, Virginia Western CC
Dale R. Bedgood, East Texas State University
Beth Beno, South Suburban College
Carolyn Bernath, Tallahassee Community College
Rebecca Berthiaume, Edison State College
William H. Beyer, University of Akron
Annette Blackwelder, Florida State University
Richelle Blair, Lakeland Community College
Kevin Bodden, Lewis and Clark College
Jeffrey Boerner, University of Wisconsin-Stout
Connie Booker, Owensboro Community and Technical College
Barry Booten, Florida Atlantic University
Laurie Boudreaux, Nicholls State University
Larry Bouldin, Roane State Community College
Bob Bradshaw, Ohlone College
Trudy Bratten, Grossmont College
Tim Bremer, Broome Community College
Tim Britt, Jackson State Community College
Holly Broesamle, Oakland CC-Auburn Hills
Michael Brook, University of Delaware
Timothy Brown, Central Washington University
Joanne Brunner, Joliet Junior College
Warren Burch, Brevard Community College
Mary Butler, Lincoln Public Schools
Melanie Butler, West Virginia University
Jim Butterbach, Joliet Junior College
Roberto Cabezas, Miami Dade College
William J. Cable, University of Wisconsin-Stevens Point
Lois Calamia, Brookdale Community College
Jim Campbell, Lincoln Public Schools
Roger Carlsen, Moraine Valley Community College
Elena Catoiu, Joliet Junior College
Mathews Chakkanakuzhi, Palomar College
Tim Chappell, Penn Valley Community College
John Collado, South Suburban College
Amy Collins, Northwest Vista College
Alicia Collins, Mesa Community College
Nelson Collins, Joliet Junior College
Rebecca Connell, Troy University
Jim Cooper, Joliet Junior College
Denise Corbett, East Carolina University
Carlos C. Corona, San Antonio College
Theodore C. Coskey, South Seattle Community College
Rebecca Connell, Troy University
Donna Costello, Plano Senior High School
Rebecca Courter, Pasadena City College
Garrett Cox, The University of Texas at San Antonio
Paul Crittenden, University of Nebraska at Lincoln
John Davenport, East Texas State University
Faye Dang, Joliet Junior College
Antonio David, Del Mar College
Stephanie Deacon, Liberty University
Duane E. Deal, Ball State University
Jerry DeGroot, Purdue North Central
Timothy Deis, University of Wisconsin-Platteville
Joanna DelMonaco, Middlesex Community College
Vivian Dennis, Eastfield College
Deborah Dillon, R. L. Turner High School
Guesna Dohrman, Tallahassee Community College
Cheryl Doolittle, Iowa State University
Karen R. Dougan, University of Florida
Jerrett Dumouchel, Florida Community College at Jacksonville
Louise Dyson, Clark College
Paul D. East, Lexington Community College
Don Edmondson, University of Texas-Austin
Erica Egizio, Joliet Junior College
Jason Eltrevoog, Joliet Junior College
Christopher Ennis, University of Minnesota
Kathy Eppler, Salt Lake Community College
Ralph Esparza, Jr., Richland College
Garret J. Etgen, University of Houston
Scott Fallstrom, Shoreline Community College
Pete Falzone, Pensacola Junior College
Arash Farahmand, Skyline College
Said Fariabli, San Antonio College
W.A. Ferguson, University of Illinois-Urbana/Champaign
Iris B. Fetta, Clemson University
Mason Flake, student at Edison Community College
Timothy W. Flood, Pittsburg State University
Robert Frank, Westmoreland County Community College
Merle Friel, Humboldt State University
Richard A. Fritz, Moraine Valley Community College
Dewey Furness, Ricks College
Mary Jule Gabiou, North Idaho College
Randy Gallaher, Lewis and Clark College
Tina Garn, University of Arizona
Dawit Getachew, Chicago State University
Wayne Gibson, Rancho Santiago College
Loran W. Gierhart, University of Texas at San Antonio and Palo Alto College
Robert Gill, University of Minnesota Duluth
Nina Girard, University of Pittsburgh at Johnstown
Sudhir Kumar Goel, Valdosta State University
Adrienne Goldstein, Miami Dade College, Kendall Campus
Joan Goliday, Sante Fe Community College
Lourdes Gonzalez, Miami Dade College, Kendall Campus
Frederic Gooding, Goucher College
Donald Goral, Northern Virginia Community College
Sue Graupner, Lincoln Public Schools
Mary Beth Grayson, Liberty University
Jennifer L. Grimsley, University of Charleston
Ken Gurganus, University of North Carolina
Igor Halfin, University of Texas-San Antonio
James E. Hall, University of Wisconsin-Madison
Judy Hall, West Virginia University
Edward R. Hancock, DeVry Institute of Technology
Julia Hassett, DeVry Institute, Dupage
Christopher Hay-Jahans, University of South Dakota
Michah Heibel, Lincoln Public Schools
LaRae Helliwell, San Jose City College
Celeste Hernandez, Richland College
Gloria P. Hernandez, Louisiana State University at Eunice
Brother Herron, Brother Rice High School
Robert Hoburg, Western Connecticut State University
Lynda Hollingsworth, Northwest Missouri State University
Deltrye Holt, Augusta State University
Charla Holzbog, Denison High School
Lee Hruby, Naperville North High School
Miles Hubbard, St. Cloud State University
Kim Hughes, California State College-San Bernardino
Stanislav, Jabuka, University of Nevada, Reno
Ron Jamison, Brigham Young University
Richard A. Jensen, Manatee Community College
Glenn Johnson, Middlesex Community College
Sandra G. Johnson, St. Cloud State University
Tuesday Johnson, New Mexico State University
Susitha Karunaratne, Purdue University North Central
Moana H. Karsteter, Tallahassee Community College
Donna Katula, Joliet Junior College
Arthur Kaufman, College of Staten Island
Thomas Kearns, North Kentucky University
Jack Keating, Massasoit Community College
Shelia Kellenbarger, Lincoln Public Schools
Rachael Kenney, North Carolina State University
Penelope Kirby, Florida State University
John B. Klassen, North Idaho College
Debra Kopcso, Louisiana State University
Lynne Kowski, Raritan Valley Community College
Yelena Kravchuk, University of Alabama at Birmingham
Ray S. Kuan, Skyline College
Keith Kuchar, Manatee Community College
Tor Kwembe, Chicago State University
Linda J. Kyle, Tarrant Country Jr. College
H.E. Lacey, Texas A & M University
Darren Lacoste, Valencia College-West Campus
Harriet Lamm, Coastal Bend College
James Lapp, Fort Lewis College
Matt Larson, Lincoln Public Schools
Christopher Lattin, Oakton Community College
Julia Ledet, Lousiana State University
Wayne Lee, St. Phillips CC
Adele LeGere, Oakton Community College
Kevin Leith, University of Houston
JoAnn Lewin, Edison College
Jeff Lewis, Johnson County Community College
Janice C. Lyon, Tallahassee Community College
Jean McArthur, Joliet Junior College
Virginia McCarthy, Iowa State University
Karla McCavit, Albion College
Michael McClendon, University of Central Oklahoma
Tom McCollow, DeVry Institute of Technology
Marilyn McCollum, North Carolina State University
Jill McGowan, Howard University
Will McGowant, Howard University
Angela McNulty, Joliet Junior College
Lisa Meads, College of the Albemarle
Laurence Maher, North Texas State University
Jay A. Malmstrom, Oklahoma City Community College
Rebecca Mann, Apollo High School
Lynn Marecek, Santa Ana College
Sherry Martina, Naperville North High School
Ruby Martinez, San Antonio College
Alec Matheson, Lamar University
Nancy Matthews, University of Oklahoma

James Maxwell, Oklahoma State University-Stillwater
Marsha May, Midwestern State University
James McLaughlin, West Chester University
Judy Meckley, Joliet Junior College
David Meel, Bowling Green State University
Carolyn Meitler, Concordia University
Samia Metwali, Erie Community College
Rich Meyers, Joliet Junior College
Eldon Miller, University of Mississippi
James Miller, West Virginia University
Michael Miller, Iowa State University
Kathleen Miranda, SUNY at Old Westbury
Chris Mirbaha, The Community College of Baltimore County
Val Mohanakumar, Hillsborough Community College
Thomas Monaghan, Naperville North High School
Miguel Montanez, Miami Dade College, Wolfson Campus
Maria Montoya, Our Lady of the Lake University
Susan Moosai, Florida Atlantic University
Craig Morse, Naperville North High School
Samad Mortabit, Metropolitan State University
Pat Mower, Washburn University
Tammy Muhs, University of Central Florida
A. Muhundan, Manatee Community College
Jane Murphy, Middlesex Community College
Richard Nadel, Florida International University
Gabriel Nagy, Kansas State University
Bill Naegele, South Suburban College
Karla Neal, Lousiana State University
Lawrence E. Newman, Holyoke Community College
Dwight Newsome, Pasco-Hernando Community College
Denise Nunley, Maricopa Community Colleges
James Nymann, University of Texas-El Paso
Mark Omodt, Anoka-Ramsey Community College
Seth F. Oppenheimer, Mississippi State University
Leticia Oropesa, University of Miami
Linda Padilla, Joliet Junior College
Sanja Pantic, University of Illinois at Chicago
E. James Peake, Iowa State University
Kelly Pearson, Murray State University
Dashamir Petrela, Florida Atlantic University
Philip Pina, Florida Atlantic University
Charlotte Pisors, Baylor University
Michael Prophet, University of Northern Iowa
Laura Pyzdrowski, West Virginia University
Carrie Quesnell, Weber State University
Neal C. Raber, University of Akron
Thomas Radin, San Joaquin Delta College
Aibeng Serene Radulovic, Florida Atlantic University
Ken A. Rager, Metropolitan State College
Traci Reed, St. Johns River State College
Kenneth D. Reeves, San Antonio College
Elsi Reinhardt, Truckee Meadows Community College
Jose Remesar, Miami Dade College, Wolfson Campus
Jane Ringwald, Iowa State University
Douglas F. Robertson, University of Minnesota, MPLS
Stephen Rodi, Austin Community College
William Rogge, Lincoln Northeast High School
Howard L. Rolf, Baylor University
Mike Rosenthal, Florida International University
Phoebe Rouse, Lousiana State University
Edward Rozema, University of Tennessee at Chattanooga
Dennis C. Runde, Manatee Community College
Paul Runnion, Missouri University of Science and Technology
Amit Saini, University of Nevada-Reno
Laura Salazar, Northwest Vista College
Alan Saleski, Loyola University of Chicago
Susan Sandmeyer, Jamestown Community College
Brenda Santistevan, Salt Lake Community College
Linda Schmidt, Greenville Technical College
Ingrid Scott, Montgomery College
A.K. Shamma, University of West Florida
Zachery Sharon, University of Texas at San Antonio
Joshua Shelor, Virginia Western CC
Martin Sherry, Lower Columbia College
Carmen Shershin, Florida International University
Tatrana Shubin, San Jose State University
Anita Sikes, Delgado Community College
Timothy Sipka, Alma College
Charlotte Smedberg, University of Tampa
Lori Smellegar, Manatee Community College
Gayle Smith, Loyola Blakefield
Cindy Soderstrom, Salt Lake Community College
Leslie Soltis, Mercyhurst College
John Spellman, Southwest Texas State University
Karen Spike, University of North Carolina
Rajalakshmi Sriram, Okaloosa-Walton Community College
Katrina Staley, North Carolina Agricultural and Technical State University
Becky Stamper, Western Kentucky University
Judy Staver, Florida Community College-South
Robin Steinberg, Pima Community College
Neil Stephens, Hinsdale South High School
Sonya Stephens, Florida A&M Univeristy
Patrick Stevens, Joliet Junior College
John Sumner, University of Tampa
Matthew TenHuisen, University of North Carolina, Wilmington
Christopher Terry, Augusta State University
Diane Tesar, South Suburban College
Tommy Thompson, Brookhaven College
Martha K. Tietze, Shawnee Mission Northwest High School
Richard J. Tondra, Iowa State University
Florentina Tone, University of West Florida
Suzanne Topp, Salt Lake Community College
Marilyn Toscano, University of Wisconsin, Superior
Marvel Townsend, University of Florida
Jim Trudnowski, Carroll College
David Tseng, Miami Dade College, Kendall Campus
Robert Tuskey, Joliet Junior College
Mihaela Vajiac, Chapman University-Orange
Julia Varbalow, Thomas Nelson Community College-Leesville
Richard G. Vinson, University of South Alabama
Jorge Viola-Prioli, Florida Atlantic University
Mary Voxman, University of Idaho
Jennifer Walsh, Daytona Beach Community College
Donna Wandke, Naperville North High School
Timothy L.Warkentin, Cloud County Community College
Melissa J. Watts, Virginia State University
Hayat Weiss, Middlesex Community College
Kathryn Wetzel, Amarillo College
Darlene Whitkenack, Northern Illinois University
Suzanne Williams, Central Piedmont Community College
Larissa Williamson, University of Florida
Christine Wilson, West Virginia University
Brad Wind, Florida International University
Anna Wiodarczyk, Florida International University
Mary Wolyniak, Broome Community College
Canton Woods, Auburn University
Tamara S. Worner, Wayne State College
Terri Wright, New Hampshire Community Technical College, Manchester
Rob Wylie, Carl Albert State College
Aletheia Zambesi, University of West Florida
George Zazi, Chicago State University
Loris Zucca, Lone Star College-Kingwood
Steve Zuro, Joliet Junior College

Michael Sullivan

Pearson would like to thank the following for their contribution to the Global Edition:

Contributors

Anuj Chatterje
Monica Sethi
Sunila Sharma, Miranda House

Reviewers

Kwa Kiam Heong, Universiti Malaya
Jairusha Jackson
Emrah Kiliç, TOBB University of Economics and Technology
Ersin Özuğurlu, Istanbul Technical University
Mani Sankar, East Point College of Engineering and Technology

Pearson MyLab

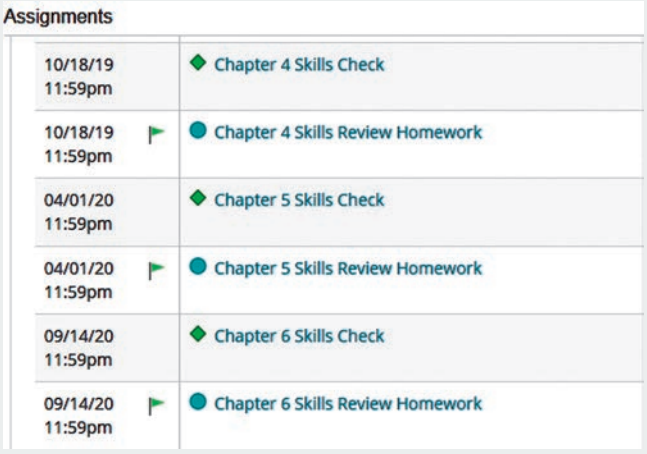
Get the Most Out of MyLab Math

Math courses are continuously evolving to help today's students succeed. It's more challenging than ever to support students with a wide range of backgrounds, learner styles, and math anxieties. The flexibility to build a course that fits instructors' individual course formats—with a variety of content options and multimedia resources all in one place—has made MyLab Math the market-leading solution for teaching and learning mathematics since its inception.

Preparedness

One of the biggest challenges in College Algebra, Trigonometry, and Precalculus is making sure students are adequately prepared with prerequisite knowledge. For a student, having the essential algebra skills upfront in this course can dramatically increase success.

- **MyLab Math with Integrated Review** can be used in corequisite courses, or simply to help students who enter without a full understanding of prerequisite skills and concepts. **Integrated Review** provides videos on review topics with a corresponding worksheet, along with premade, assignable skills-check quizzes and personalized review homework assignments. **Integrated Review** is now available within all Sullivan 11th Edition MyLab Math courses.



https://mlm.pearson.com/global/

Resources for Success

Pearson MyLab

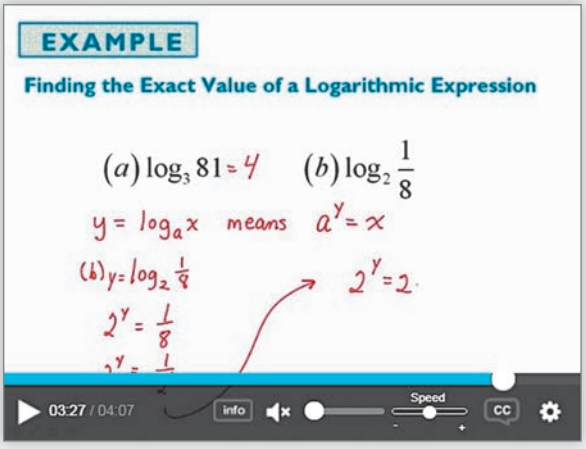
MyLab Math Online Course for Precalculus, 11th Edition by Michael Sullivan (access code required)

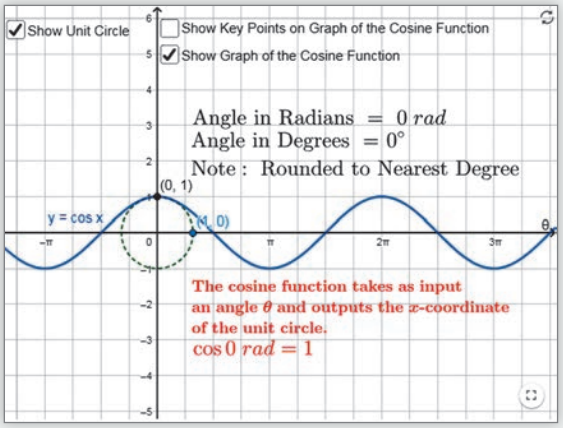
MyLab™ Math is tightly integrated with each author's style, offering a range of author-created multimedia resources, so your students have a consistent experience.

Video Program and Resources

Author in Action Videos are actual classroom lectures by Michael Sullivan III with fully worked-out examples.

- **Video assessment** questions are available to assign in MyLab Math for key videos.
- **Updated!** The corresponding **Guided Lecture Notes** assist students in taking thorough, organized, and understandable notes while watching Author in Action videos.



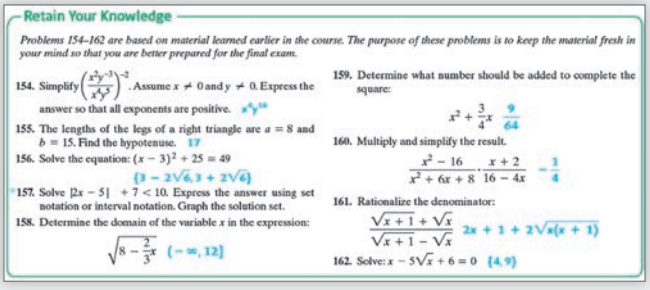


Guided Visualizations

New! Guided Visualizations, created in GeoGebra by Michael Sullivan III, bring mathematical concepts to life, helping students visualize the concept through directed exploration and purposeful manipulation. Assignable in MyLab Math with assessment questions to check students' conceptual understanding.

Retain Your Knowledge Exercises

Updated! Retain Your Knowledge Exercises, assignable in MyLab Math, improve students' recall of concepts learned earlier in the course. New for the 11th Edition, additional exercises will be included that will have an emphasis on content that students will build upon in the immediate upcoming section.



https://mlm.pearson.com/global/

Resources for Success

Pearson MyLab

Instructor Resources

Online resources can be downloaded from the Instructor Resource Center.

Instructor's Solutions Manual

Includes fully worked solutions to all exercises in the text.

Learning Catalytics Question Library

Questions written by Michael Sullivan III are available within MyLab Math to deliver through Learning Catalytics to engage students in your course.

Powerpoint® Lecture Slides

Fully editable slides correlate to the textbook.

Mini Lecture Notes

Includes additional examples and helpful teaching tips, by section.

Testgen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Online Chapter Projects

Additional projects that give students an opportunity to apply what they learned in the chapter.

Student Resources

Additional resources to enhance student success.

Lecture Video

Author in Action videos are actual classroom lectures with fully worked out examples presented by Michael Sullivan, III. All video is assignable within MyLab Math.

Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all chapter test exercises from the text. These are available in MyLab Math and on YouTube.

Guided Lecture Notes

These lecture notes assist students in taking thorough, organized, and understandable notes while watching Author in Action videos. Students actively participate in learning the how/why of important concepts through explorations and activities. The Guided Lecture Notes are available as PDF's and customizable Word files in MyLab Math. They can also be packaged with the text and the MyLab Math access code.

Applications Index

Graphs

1

How to Value a House

Two things to consider in valuing a home: (1) How does it compare to similar nearby homes that have sold recently? (2) What value do you place on the advertised features and amenities?

The Zestimate® home value is a good starting point in figuring out the value of a home. It shows you how the home compares relative to others in the area, but you then need to add in all the other qualities that only someone who has seen the house knows.

Looking at "comps"

Knowing whether an asking price is fair will be important when you're ready to make an offer on a house. It will be even more important when your mortgage lender hires an appraiser to determine whether the house is worth the loan you're after.

Check on Zillow to see recent sales of similar, or comparable, homes in the area. Print them out and keep these "comps." You'll be referring to them quite a bit.

Note that "recent sales" usually means within the past six months. A sales price from a year ago probably bears little or no relation to what is going on in your area right now. In fact, some lenders will not accept comps older than three months.

Market activity also determines how easy or difficult it is to find accurate comps. In a "hot" or busy market, you're likely to have lots of comps to choose from. In a less active market finding reasonable comps becomes harder. And if the home you're looking at has special design features, finding a comparable property is harder still. It's also necessary to know what's going on in a given sub-segment. Maybe large, high-end homes are selling like hotcakes, but owners of smaller houses are staying put, or vice versa.

***Source**: http://luthersanchez.com/2016/03/09/how-to-value-a-house/*

—See the Internet-based Chapter Project—

A Look Back

Appendix A reviews skills from intermediate algebra.

A Look Ahead

Here we connect algebra and geometry using the rectangular coordinate system. In the 1600s, algebra had developed to the point that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) were able to use rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This enabled both geometers and algebraists to gain new insights into their subjects, which had been thought to be separate but now were seen as connected.

Outline

1.1 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Section A.1, pp. A1–A10)
- Geometry Essentials (Section A.2, pp. A14–A19)

Now Work the **'Are You Prepared?'** problems on page 42.

OBJECTIVES 1 Use the Distance Formula (p. 39)
2 Use the Midpoint Formula (p. 41)

Rectangular Coordinates

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the ***x*-axis**, the vertical line the ***y*-axis**, and the point of intersection the **origin *O***. See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. In mathematics, we usually use the same scale on each axis, but in applications, different scales appropriate to the application may be used.

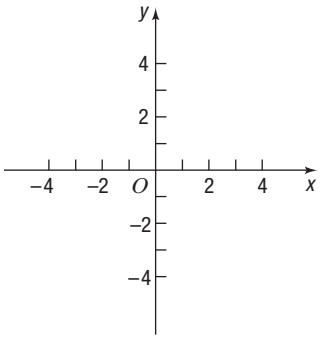


Figure 1 *xy*-Plane

The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system**. The x-axis and y-axis lie in a *plane* called the ***xy*-plane**, and the x-axis and y-axis are referred to as the **coordinate axes**.

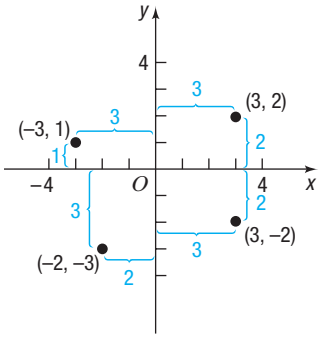


Figure 2

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, gives us enough information to locate the point P in the plane.

For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the ***x*-coordinate**, or **abscissa**, of P, and y is the ***y*-coordinate**, or **ordinate**, of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$. Usually, we will simply say "the point (x, y)" rather than "the point whose coordinates are (x, y)."

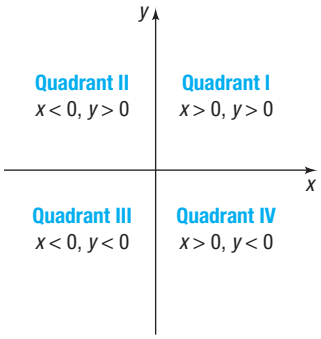


Figure 3

The coordinate axes partition the xy-plane into four sections called **quadrants**, as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

Now Work PROBLEM 15

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

COMMENT On a graphing calculator, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle**. See Figure 4 for a typical viewing rectangle. You should now read Section B.1, *The Viewing Rectangle*.

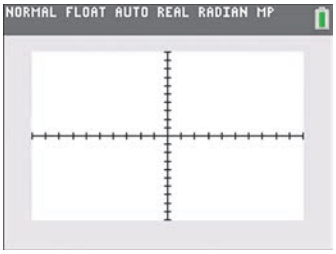


Figure 4 TI-84 Plus C Standard Viewing Rectangle ■

1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the x-axis and y-axis, then all distances in the xy-plane can be measured using this unit of measurement.

EXAMPLE 1 **Finding the Distance between Two Points**

Find the distance d between the points $(1, 3)$ and $(5, 6)$.

Solution

Need to Review?
The Pythagorean Theorem and its converse are discussed in Section A.2, pp. A14–A15.

First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a line segment. See Figure 5(a). To find the length d, begin by drawing a horizontal line segment from $(1,3)$ to $(5,3)$ and a vertical line segment from $(5,3)$ to $(5,6)$, forming a right triangle, as shown in Figure 5(b). One leg of the triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance d that we seek is

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$

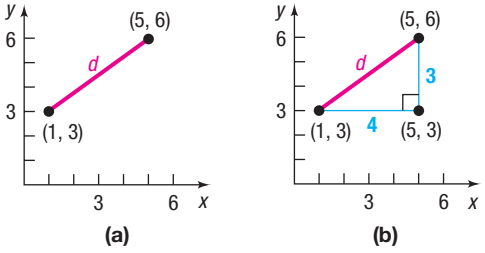


Figure 5

The **distance formula** provides a straightforward method for computing the distance between two points.

THEOREM Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

In Words
To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

Proof of the Distance Formula Let (x_1, y_1) denote the coordinates of point P_1 and let (x_2, y_2) denote the coordinates of point P_2.

- Assume that the line joining P_1 and P_2 is neither horizontal nor vertical. Refer to Figure 6(a) on the next page. The coordinates of P_3 are (x_2, y_1). The horizontal

distance from P_1 to P_3 equals the absolute value of the difference of the x-coordinates, $|x_2 - x_1|$. The vertical distance from P_3 to P_2 equals the absolute value of the difference of the y-coordinates, $|y_2 - y_1|$. See Figure 6(b). The distance $d(P_1, P_2)$ is the length of the hypotenuse of the right triangle, so, by the Pythagorean Theorem, it follows that

$$\begin{aligned}[d(P_1, P_2)]^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\end{aligned}$$

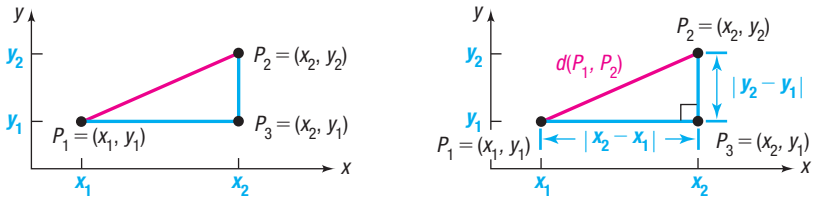


Figure 6

- If the line joining P_1 and P_2 is horizontal, then the y-coordinate of P_1 equals the y-coordinate of P_2; that is, $y_1 = y_2$. Refer to Figure 7(a). In this case, the distance formula (1) still works, because for $y_1 = y_2$, it reduces to

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + 0^2} = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$$

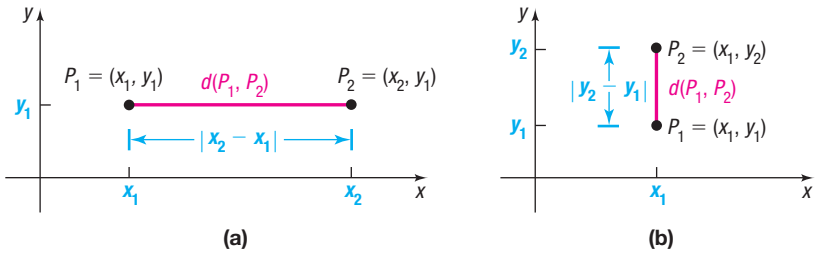


Figure 7

- A similar argument holds if the line joining P_1 and P_2 is vertical. See Figure 7(b). ■

EXAMPLE 2 Using the Distance Formula

Find the distance d between the points $(-4, 5)$ and $(3, 2)$.

Solution Using the distance formula, equation (1), reveals that the distance d is

$$\begin{aligned}d &= \sqrt{[3 - (-4)]^2 + (2 - 5)^2} = \sqrt{7^2 + (-3)^2} \\ &= \sqrt{49 + 9} = \sqrt{58} \approx 7.62\end{aligned}$$

Now Work PROBLEMS 19 AND 23

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Also, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. And, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from P_1 to P_2 or from P_2 to P_1; that is, $d(P_1, P_2) = d(P_2, P_1)$.

The introduction to this chapter mentioned that rectangular coordinates enable us to translate geometry problems into algebra problems, and vice versa. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

EXAMPLE 3 **Using Algebra to Solve a Geometry Problem**

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle ABC.
(b) Find the length of each side of the triangle.
(c) Show that the triangle is a right triangle.
(d) Find the area of the triangle.

Solution

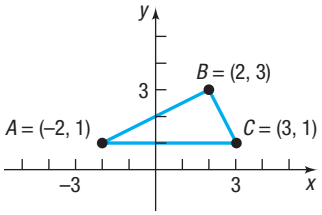


Figure 8

(a) Figure 8 shows the points A, B, C and the triangle ABC.

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$d(A, B) = \sqrt{[2 - (-2)]^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$
$$d(B, C) = \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$$
$$d(A, C) = \sqrt{[3 - (-2)]^2 + (1 - 1)^2} = \sqrt{25 + 0} = 5$$

(c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 8, it seems reasonable to conjecture that the angle at vertex B might be a right angle. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

Using the results in part (b) yields

$$[d(A, B)]^2 + [d(B, C)]^2 = \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2$$
$$= 20 + 5 = 25 = [d(A, C)]^2$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

(d) Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 2\sqrt{5} \cdot \sqrt{5} = 5 \text{ square units}$$

Now Work PROBLEM 33

2 Use the Midpoint Formula

We now derive a formula for the coordinates of the **midpoint of a line segment**. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. See Figure 9. The triangles P_1AM and MBP_2 are congruent. [Do you see why? $d(P_1, M) = d(M, P_2)$ is given; also, $\angle AP_1M = \angle BMP_2$* and $\angle P_1MA = \angle MP_2B$. So, we have angle–side–angle.] Because triangles P_1AM and MBP_2 are congruent, corresponding sides are equal in length. That is,

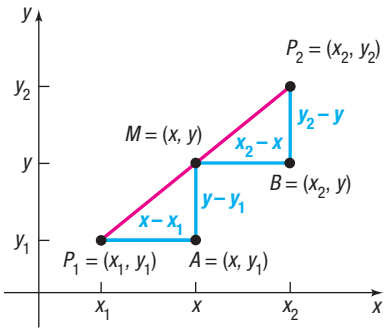


Figure 9

$$x - x_1 = x_2 - x \quad \text{and} \quad y - y_1 = y_2 - y$$
$$2x = x_1 + x_2 \qquad 2y = y_1 + y_2$$
$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

In Words
To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.

THEOREM Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \textbf{(2)}$$

EXAMPLE 4 **Finding the Midpoint of a Line Segment**

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

Solution

Use the midpoint formula (2) with $x_1 = -5, y_1 = 5, x_2 = 3$, and $y_2 = 1$. The coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 10.

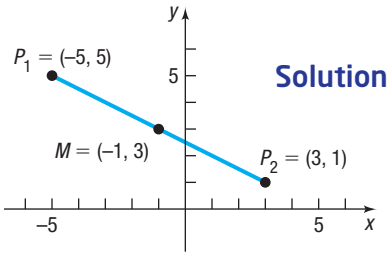


Figure 10

Now Work PROBLEM 39

1.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line, the origin is assigned the number _____. (p. A4)
2. If -3 and 5 are the coordinates of two points on the real number line, the distance between these points is _____. (pp. A5–A6)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is _____. (p. A14)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A15)
5. The area A of a triangle whose base is b and whose altitude is h is $A =$ _____. (p. A15)
6. ***True or False*** Two triangles are congruent if two angles and the included side of one equals two angles and the included side of the other. (pp. A16–A17)

Concepts and Vocabulary

7. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the _____ of P, and y is the _____ of P.
8. The coordinate axes partition the xy-plane into four sections called _____.
9. If three distinct points P, Q, and R all lie on a line, and if $d(P, Q) = d(Q, R)$, then Q is called the _____ of the line segment from P to R.
10. ***True or False*** The distance between two points is sometimes a negative number.
11. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.
12. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.
13. ***Multiple Choice*** Which of the following statements is true for a point (x, y) that lies in quadrant III?
    (a) Both x and y are positive.
    (b) Both x and y are negative.
    (c) x is positive, and y is negative.
    (d) x is negative, and y is positive.
14. ***Multiple Choice*** Choose the expression that equals the distance between two points (x_1, y_1) and (x_2, y_2).
    (a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    (b) $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$
    (c) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
    (d) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

Skill Building

In Problems 15 and 16, plot each point in the xy-plane. State which quadrant or on what coordinate axis each point lies.

15. **(a)** $A = (-3, 2)$ **(b)** $B = (6, 0)$ **(c)** $C = (-2, -2)$ **(d)** $D = (6, 5)$ **(e)** $E = (0, -3)$ **(f)** $F = (6, -3)$

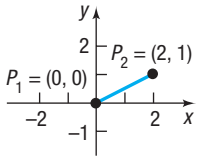
16. **(a)** $A = (1, 4)$ **(b)** $B = (-3, -4)$ **(c)** $C = (-3, 4)$ **(d)** $D = (4, 1)$ **(e)** $E = (0, 1)$ **(f)** $F = (-3, 0)$

17. Plot the points $(0, 3), (1, 3), (-2, 3), (5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

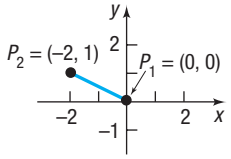
18. Plot the points $(2, 0), (2, -3), (2, 4), (2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.

In Problems 19–32, find the distance d between the points P_1 and P_2.

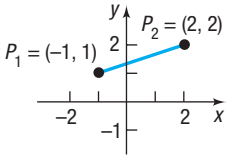
19.



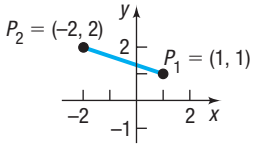
20.



21.



22.



23. $P_1 = (3, -4)$; $P_2 = (5, 4)$

24. $P_1 = (-1, 0)$; $P_2 = (2, 4)$

25. $P_1 = (2, -3)$; $P_2 = (4, 2)$

26. $P_1 = (-7, 3)$; $P_2 = (4, 0)$

27. $P_1 = (-4, -3)$; $P_2 = (6, 2)$

28. $P_1 = (5, -2)$; $P_2 = (6, 1)$

29. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

30. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

31. $P_1 = (a, a)$; $P_2 = (0, 0)$

32. $P_1 = (a, b)$; $P_2 = (0, 0)$

In Problems 33–38, plot each point and form the triangle ABC. Show that the triangle is a right triangle. Find its area.

33. $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$

34. $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$

35. $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$

36. $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$

37. $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

38. $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$

In Problems 39–46, find the midpoint of the line segment joining the points P_1 and P_2.

39. $P_1 = (3, -4)$; $P_2 = (5, 4)$

40. $P_1 = (-2, 0)$; $P_2 = (2, 4)$

41. $P_1 = (2, -3)$; $P_2 = (4, 2)$

42. $P_1 = (-1, 4)$; $P_2 = (8, 0)$

43. $P_1 = (-4, -3)$; $P_2 = (2, 2)$

44. $P_1 = (7, -5)$; $P_2 = (9, 1)$

45. $P_1 = (a, a)$; $P_2 = (0, 0)$

46. $P_1 = (a, b)$; $P_2 = (0, 0)$

Applications and Extensions

47. If the point $(3, 8)$ is shifted 2 units to the right and 4 units down, what are its new coordinates?

48. If the point $(-1, 6)$ is shifted 2 units to the left and 4 units up, what are its new coordinates?

49. Find all points having an x-coordinate of 4 whose distance from the point $(-4, 2)$ is 10.

(a) By using the Pythagorean Theorem.
(b) By using the distance formula.

50. Find all points having a y-coordinate of -6 whose distance from the point $(1, 2)$ is 17.

(a) By using the Pythagorean Theorem.
(b) By using the distance formula.

51. Find all points on the x-axis that are 12 units from the point $(5, -6)$.

52. Find all points on the y-axis that are 6 units from the point $(4, -3)$.

53. Suppose that $A = (2, 5)$ are the coordinates of a point in the xy-plane.

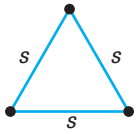
(a) Find the coordinates of the point if A is shifted 2 units to the right and 3 units down.
(b) Find the coordinates of the point if A is shifted 1 unit to the left and 6 units up.

54. Plot the points $A = (-1, 8)$ and $M = (2, 3)$ in the xy-plane. If M is the midpoint of a line segment AB, find the coordinates of B.

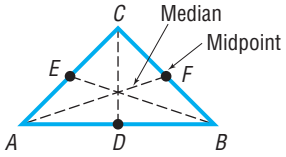
55. The midpoint of the line segment from P_1 to P_2 is $(-6, 5)$. If $P_1 = (-6, 3)$. what is P_2?

56. The midpoint of the line segment from P_1 to P_2 is $(5, -4)$. If $P_2 = (7, -2)$, what is P_1?

57. Geometry An **equilateral triangle** has three sides of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$ find the third vertex. How many of these triangles are possible?



58. Geometry The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.



*In Problems 59–62, find the length of each side of the triangle determined by the three points P_1, P_2, and P_3. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An **isosceles triangle** is one in which at least two of the sides are of equal length.)*

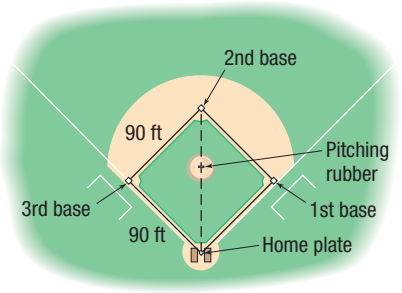
59. $P_1 = (-1, 4)$; $P_2 = (6, 2)$; $P_3 = (4, -5)$

60. $P_1 = (4, 2)$; $P_2 = (10, 4)$; $P_3 = (6, -4)$

61. $P_1 = (7, 2)$; $P_2 = (-4, 0)$; $P_3 = (4, 6)$

62. $P_1 = (-8, -3)$, $P_2 = (0, 15)$, $P_3 = (5, 2)$

63. Baseball A major league baseball "diamond" is actually a square 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



64. Little League Baseball The layout of a Little League playing field is a square 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?

***Source**: 2018 Little League Baseball Official Regulations, Playing Rules, and Operating Policies*

65. Baseball Refer to Problem 63. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at (310, 15) how far is it from the right fielder to second base?

(c) If the center fielder is located at (300, 300), how far is it from the center fielder to third base?

66. Little League Baseball Refer to Problem 64. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

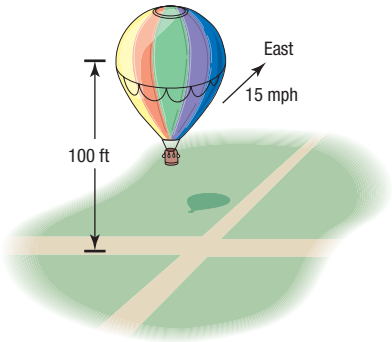
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at (180, 20), how far is it from the right fielder to second base?

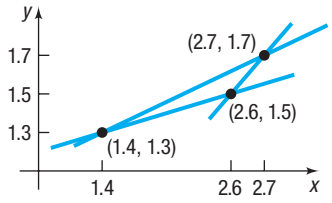
(c) If the center fielder is located at (220, 220), how far is it from the center fielder to third base?

67. Distance between Moving Objects A Ford Focus and a Freightliner Cascadia truck leave an intersection at the same time. The Focus heads east at an average speed of 60 miles per hour, while the Cascadia heads south at an average speed of 45 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.

68. Distance of a Moving Object from a Fixed Point A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance d (measured in feet) from the balloon to the intersection t seconds later.



69. Drafting Error When a draftsman draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle**. If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure shows one such error triangle.

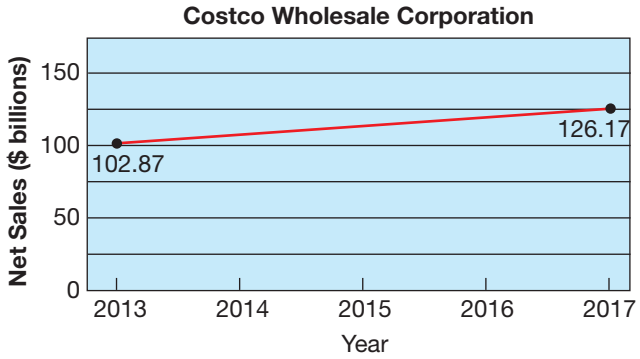


(a) Find an estimate for the desired intersection point.

(b) Find the distance from (1.4, 1.3) to the midpoint found in part (a).

70. Net Sales The figure illustrates the net sales growth of Costco Wholesale Corporation from 2013 through 2017. Use the midpoint formula to estimate the net sales of Costco Wholesale Corporation in 2015. How does your result compare to the reported value of $113.67 billion?

***Source**: Costco Wholesale Corporation 2017 Annual Report*



71. Poverty Threshold A poverty threshold represents the minimum annual household income for a family not to be considered poor. In 1995, the poverty threshold for a family of four with two children under the age of 18 years was $15,598. In 2005, the poverty threshold for a family of four with two children under the age of 18 years was $19,508. Assuming poverty thresholds increase in a straight-line fashion, use the midpoint formula to estimate the poverty threshold of a family of four with two children under the age of 18 in 2000.

72. *Challenge Problem* Geometry Verify that the points $(0,0), (a,0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle.

73. *Challenge Problem* Geometry A point P is equidistant from $(-5, 1)$ and $(4, -4)$. Find the coordinates of P if its y-coordinate is twice its x-coordinate.

74. *Challenge Problem* Geometry Find the midpoint of each diagonal of a square with side of length s. Draw the conclusion that the diagonals of a square intersect at their midpoints.

[**Hint**: Use $(0, 0)$, $(0, s)$, $(s, 0)$, and (s, s) as the vertices of the square.]

75. *Challenge Problem* Geometry For any parallelogram, prove that the sum of the squares of the lengths of the sides equals the sum of the squares of the lengths of the diagonals.

[**Hint**: Use $(0, 0)$, $(a, 0)$, $(a + b, c)$, and (b, c) as the vertices of the parallelogram. Assume a, b, and c are positive.]

Explaining Concepts: Discussion and Writing

76. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms "coordinate axes," "ordered pair," "coordinates," "plot," "x-coordinate," and "y-coordinate."

'Are You Prepared?' Answers

1. 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$ **5.** $\frac{1}{2}bh$ **6.** True

1.2 Graphs of Equations in Two Variables; Intercepts; Symmetry

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Linear Equations (Section A.6, pp. A44–A45)
- Solve a Quadratic Equation by Factoring (Section A.6, pp. A47–A48)

Now Work the **'Are You Prepared?'** problems on page 53.

OBJECTIVES

1 Graph Equations by Plotting Points

An **equation in two variables**, say x and y, is a statement in which two expressions involving x and y are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the value of the variables. Any values of x and y that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables x and y:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$

The first of these, $x^2 + y^2 = 5$, is satisfied for $x = 1, y = 2$, since $1^2 + 2^2 = 5$. Other choices of x and y, such as $x = -1, y = -2$, also satisfy this equation. It is not satisfied for $x = 2$ and $y = 3$, since $2^2 + 3^2 = 4 + 9 = 13 \neq 5$.

The **graph of an equation in two variables** x and y consists of the set of points in the xy-plane whose coordinates (x, y) satisfy the equation.

Graphs play an important role in helping us to visualize the relationships that exist between two variables or quantities. Table 1 shows the average price of gasoline in the United States for the years 1991–2017 (adjusted for inflation). If we plot these data using year as the x-coordinate and price as the y-coordinate, and then connect the points (year, price), we obtain Figure 11.

Table 1 Average Price of Gasoline

Year	Price	Year	Price	Year	Price
1991	1.98	2000	2.11	2009	2.68
1992	1.90	2001	1.97	2010	3.12
1993	1.81	2002	1.83	2011	3.84
1994	1.78	2003	2.07	2012	3.87
1995	1.78	2004	2.40	2013	3.68
1996	1.87	2005	2.85	2014	3.48
1997	1.83	2006	3.13	2015	2.51
1998	1.55	2007	3.31	2016	2.19
1999	1.67	2008	3.70	2017	2.38

Source: U.S. Energy Information Administration *(https://www.eia.gov/dnav/pet/pet_pri_gnd_dcus_nus_a.htm)*

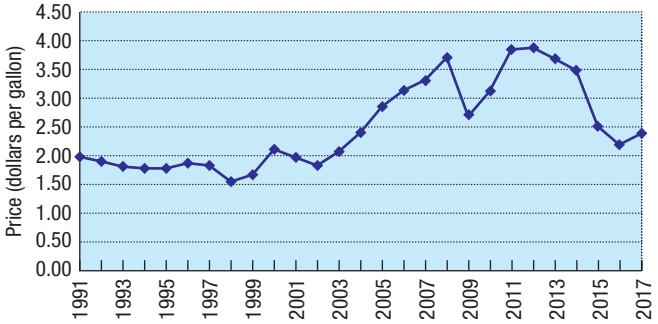


Figure 11

EXAMPLE 1 **Determining Whether a Point Is on the Graph of an Equation**

Determine whether the points are on the graph of the equation $2x - y = 6$.

(a) $(2, 3)$ (b) $(2, -2)$

Solution

(a) For the point $(2, 3)$, check to see whether $x = 2, y = 3$ satisfies the equation $2x - y = 6$.

$$2x - y = 2 \cdot 2 - 3 = 4 - 3 = 1 \neq 6$$

The equation is not satisfied, so the point $(2, 3)$ is not on the graph of $2x - y = 6$.

(b) For the point $(2, -2)$,

$$2x - y = 2 \cdot 2 - (-2) = 4 + 2 = 6$$

The equation is satisfied, so the point $(2, -2)$ is on the graph of $2x - y = 6$.

Now Work PROBLEM 13

EXAMPLE 2 Graphing an Equation by Plotting Points

Graph the equation: $y = 2x + 5$

Solution

The graph consists of all points (x, y) that satisfy the equation. To locate some of these points (and get an idea of the pattern of the graph), assign some numbers to x, and find corresponding values for y.

If	Then	Point on Graph
$x = 0$	$y = 2 \cdot 0 + 5 = 5$	$(0, 5)$
$x = 1$	$y = 2 \cdot 1 + 5 = 7$	$(1, 7)$
$x = -5$	$y = 2 \cdot (-5) + 5 = -5$	$(-5, -5)$
$x = 10$	$y = 2 \cdot 10 + 5 = 25$	$(10, 25)$

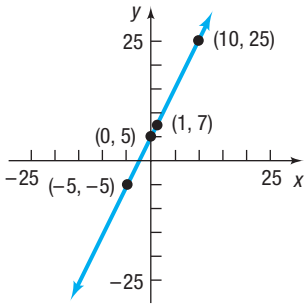


Figure 12 $y = 2x + 5$

By plotting these points and then connecting them, we obtain the graph (a *line*) of the equation $y = 2x + 5$, as shown in Figure 12.

EXAMPLE 3 Graphing an Equation by Plotting Points

Graph the equation: $y = x^2$

Solution

Table 2 provides several points on the graph of $y = x^2$. Plotting these points and connecting them with a smooth curve gives the graph (a *parabola*) shown in Figure 13.

Table 2

x	$y = x^2$	(x, y)
−4	16	(−4, 16)
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)
4	16	(4, 16)

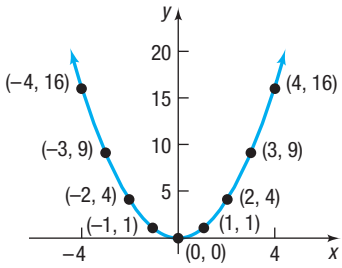


Figure 13 $y = x^2$

The graphs of the equations shown in Figures 12 and 13 do not show all points. For example, in Figure 12, the point (20, 45) is a part of the graph of $y = 2x + 5$, but it is not shown. Since the graph of $y = 2x + 5$ can be extended out indefinitely, we use arrows to indicate that the pattern shown continues. It is important, when showing a graph, to present enough of the graph so that any viewer of the illustration will "see" the rest of it as an obvious continuation of what is actually there. This is referred to as a **complete graph**.

One way to obtain the complete graph of an equation is to plot enough points on the graph for a pattern to become evident. Then these points are connected with a smooth curve following the suggested pattern. But how many points are sufficient? Sometimes knowledge about the equation tells us. For example, we will learn in the next section that if an equation is of the form $y = mx + b$, then its graph is a line. In this case, only two points are needed to obtain the complete graph.

COMMENT Another way to obtain the graph of an equation is to use a graphing utility. Read Section B.2, *Using a Graphing Utility to Graph Equations*. ■

One purpose of this text is to investigate the properties of equations in order to decide whether a graph is complete. Sometimes we shall graph equations by plotting points. Shortly, we shall investigate various techniques that will enable us to graph an equation without plotting so many points.

Two techniques that sometimes reduce the number of points required to graph an equation involve finding *intercepts* and checking for *symmetry*.

2 Find Intercepts from a Graph

The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts** of the graph. See Figure 14. The x-coordinate of a point at which the graph crosses or touches the x-axis is an ***x*-intercept**, and the y-coordinate of a point at which the graph crosses or touches the y-axis is a ***y*-intercept**.

In Words
Intercepts are points (ordered pairs). An x-intercept or a y-intercept is a number. For example, the point (3, 0) is an intercept; the number 3 is an x-intercept.

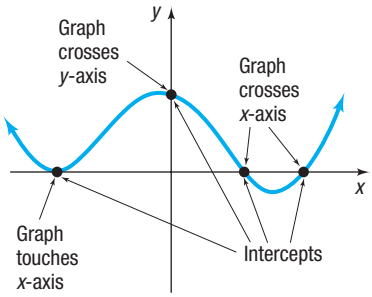


Figure 14

EXAMPLE 4 Finding Intercepts from a Graph

Find the intercepts of the graph in Figure 15. What are its x-intercepts? What are its y-intercepts?

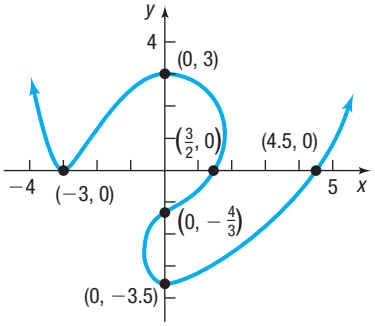


Figure 15

Solution The intercepts of the graph are the points

$$(-3, 0), \quad (0, 3), \quad \left(\frac{3}{2}, 0\right), \quad \left(0, -\frac{4}{3}\right), \quad (0, -3.5), \quad (4.5, 0)$$

The x-intercepts are -3, $\frac{3}{2}$, and 4.5; the y-intercepts are -3.5, $-\frac{4}{3}$, and 3. ⌟

In Example 4, notice that intercepts are listed as ordered pairs, and the x-intercepts and the y-intercepts are listed as numbers. We use this distinction throughout the text.

Now Work PROBLEM 41(a)

3 Find Intercepts from an Equation

The intercepts of a graph can be found from its equation by using the fact that points on the x-axis have y-coordinates equal to 0, and points on the y-axis have x-coordinates equal to 0.

COMMENT For many equations, finding intercepts may not be so easy. In such cases, a graphing utility can be used. Read the first part of Section B.3, *Using a Graphing Utility to Locate Intercepts and Check for Symmetry*, to find out how to locate intercepts using a graphing utility. ■

Procedure for Finding Intercepts

- To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x, where x is a real number.
- To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y, where y is a real number.

EXAMPLE 5 Finding Intercepts from an Equation

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$. Then graph $y = x^2 - 4$ by plotting points.

Solution To find the x-intercept(s), let $y = 0$ and obtain the equation

$$\begin{aligned} x^2 - 4 &= 0 && \text{y = x² − 4 with y = 0} \\ (x + 2)(x - 2) &= 0 && \text{Factor.} \\ x + 2 = 0 \quad &\text{or} \quad x - 2 = 0 && \text{Use the Zero-Product Property.} \\ x = -2 \quad &\text{or} \quad x = 2 && \text{Solve.} \end{aligned}$$

The equation has two solutions, -2 and 2. The x-intercepts are -2 and 2.

To find the y-intercept(s), let $x = 0$ in the equation.

$$\begin{aligned} y &= x^2 - 4 \\ &= 0^2 - 4 = -4 \end{aligned}$$

The y-intercept is -4.

Since $x^2 \geq 0$ for all x, we deduce from the equation $y = x^2 - 4$ that $y \geq -4$ for all x. This information, the intercepts, and the points from Table 3 enable us to graph $y = x^2 - 4$. See Figure 16.

Table 3

x	$y = x^2 - 4$	(x, y)
-3	5	$(-3, 5)$
-1	-3	$(-1, -3)$
1	-3	$(1, -3)$
3	5	$(3, 5)$

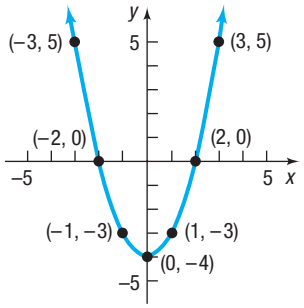


Figure 16 $y = x^2 - 4$

Now Work PROBLEM 23

4 Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin

Another helpful tool for graphing equations by hand involves *symmetry*, particularly symmetry with respect to the x-axis, the y-axis, and the origin.

Symmetry often occurs in nature. Consider the picture of the butterfly. Do you see the symmetry?

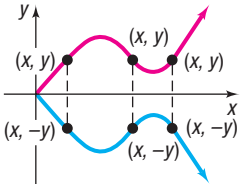


Figure 17 Symmetry with respect to the x-axis

DEFINITION Symmetry with Respect to the x-Axis

A graph is **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

Figure 17 illustrates the definition. Note that when a graph is symmetric with respect to the x-axis, the part of the graph above the x-axis is a reflection (or mirror image) of the part below it, and vice versa.

EXAMPLE 6 **Points Symmetric with Respect to the x-Axis**

If a graph is symmetric with respect to the x-axis, and the point $(3, 2)$ is on the graph, then the point $(3, -2)$ is also on the graph.