GLOBAL EDITION

Using and Understanding Mathematics

A Quantitative Reasoning Approach

SIXTH EDITION

Jeffrey Bennett • William Briggs

ALWAYS LEARNING

PEARSON

Math for College, Career, and Life

We use math in our day-to-day lives even when we don't realize it. The goal of this book is to increase mathematical literacy so we use it more effectively in everyday life. Mathematics can help us to understand a variety of topics and issues, making us more aware of both the uses and abuses of math. The ultimate goal is to become better educated citizens and be successful in our college experiences, our careers, and our lives.

Each chapter offers an **Activity** designed to spur discussion of some interesting facet of the topics covered in the chapter. [p. 314, 5A]

ACTIVITY

Cell Phones and Driving

Use this activity to gain a sense of the kinds of problems this chapter will enable you to study.

Is it safe to use a cell phone while driving? The science of statistics provides a way to approach this question, and the results of many studies indicate that the answer is no. The National Safety Council estimates that approximately 1.6 million car crashes each year (more than a quarter of the total) are caused by some type of distraction, most commonly the use of a cell phone for talking or texting. In fact, some studies suggest that merely talking on a cell phone makes you as dangerous as a drunk driver. As preparation for your study of statistics in this chapter, work individually or in groups to research the issues raised in the following questions. Discuss your findings.

IN YOUR WORLD

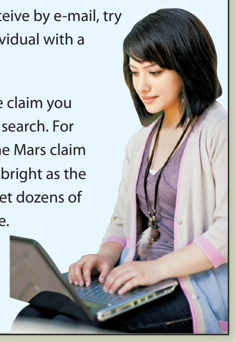
Web Searches to Verify Web Sources

While some information on the Web is inaccurate or biased, the Web is also a great source for checking the accuracy of information. A good way to start is with "fact checking" websites, as long as you also verify that the fact checkers have a reputation for fairness and accuracy. A few reputable fact-checking sites include:

rg, supported by the non-
lic Policy Center; PolitiFact.
he Fact Checker," a blog
e.
dd claims, Snopes.com

- To check the validity of messages you receive by e-mail, try TruthOrFiction.com, run by a private individual with a reputation for fairness and accuracy.

If none of those sources has covered the claim you are investigating, try a plain language Web search. For example, if you type the first sentence of the Mars claim ("On August 27, Mars will look as large and bright as the full Moon . . .") into a search engine, you'll get dozens of hits that discuss the claim and why it is false. Of course, if your search turns up conflicting claims about accuracy, you'll still need to decide which claims to believe.



IN YOUR WORLD

47. **Political Action.** This unit outlined numerous budgetary problems facing the U.S. government, as they stood at the time the book was written. Has there been any significant political action to deal with any of these problems? Learn what, if anything, has changed over the past couple of years, then write a one-page position paper outlining your own recommendations for the future.
48. **Debt Problem.** How serious a problem is the gross debt? Find arguments on both sides of this question. Summarize the arguments, and state your own opinion.

In Your World boxes focus on topics that students are likely to encounter in the world around them, whether in the news, in consumer decisions, or in political discussions. This is further enhanced with In Your World exercises, designed to spur additional research or discussion that will help students relate the unit content to the themes of college, careers, and life. [p. 309, 4F and p. 39, 1A,]

Does It Make Sense? questions test conceptual understanding by asking students to decide whether the given statements are sensible and to explain why or why not. These questions encourage students to stop and think critically about a problem rather than just focusing on getting an answer. [p. 380, 5E]

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. There is a strong negative correlation between the price of tickets and the number of tickets sold. This suggests that if we want to sell a lot of tickets, we should lower the price.
8. There is a strong positive correlation between the amount of time spent studying and grades in mathematics classes. This suggests that if you want to get a good grade, you should spend more time studying.

Why Should You Care About Quantitative Reasoning?

Quantitative reasoning is the ability to interpret and reason with information that involves numbers or mathematical ideas. It is a crucial aspect of literacy, and it is essential in making important decisions and understanding contemporary issues.

The topics covered in this text will help you work with quantitative information and make critical decisions. For example:

- You should possess strong skills in critical and logical thinking so that you can make wise personal decisions, navigate the media, and be an informed citizen. For example, do you know why you'd end up behind if you accepted a temporary 10% pay cut now and then received a 10% pay raise later? This particular question is covered in Unit 3A, but throughout the book you'll learn how to evaluate quantitative questions on topics ranging from personal decisions to major global issues.
- You should have a strong number sense and be proficient at estimation so that you can put numbers from the news into a context that makes them understandable. For example, do you know how to make sense of the more than $17 trillion federal debt? Unit 3B discusses how you can put such huge numbers in perspective, and Unit 4F discusses how the federal debt grew so large.
- You should possess the mathematical tools needed to make basic financial decisions. For example, do you enjoy a latte every morning before class? Sometimes two? Unit 4A explores how such a seemingly harmless habit can drain more than $2400 from your wallet every year.
- You should be able to read news reports of statistical studies in a way that will allow you to evaluate them critically and decide whether and how they should affect your personal beliefs. For example, how should you decide whether a new opinion poll accurately reflects the views of Americans? Chapter 5 covers the basic concepts that lie behind the statistical studies and graphics you'll see in the news, and discusses how you can decide for yourself whether you should believe a statistical study.
- You should be familiar with basic ideas of probability and risk and be aware of how they affect your life. For example, would you pay $20,000 for a product that, over 20 years, will kill nearly as many people as live in San Francisco? In Unit 7D, you'll see that the answer is very likely yes—just one of many surprises that you'll encounter as you study probability in Chapter 7.
- You should understand how mathematics helps us study important social issues, such as global warming, the growth of populations, the depletion of resources, apportionment of Congressional representatives, and methods of voting. For example, Unit 12D discusses the nature of redistricting and how gerrymandering has made congressional elections less competitive than they might otherwise be.

In sum, this text will focus on understanding and interpreting mathematical topics to help you develop the quantitative reasoning skills you will need for college, career, and life.

6TH EDITION

Using & Understanding MATHEMATICS

A Quantitative Reasoning Approach

Global Edition

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PEARSON

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This book is dedicated to everyone who wants a better understanding of our world, and especially to those who have struggled with mathematics in the past. We hope this book will help you achieve your goals.

And it is dedicated to those who make our own lives brighter, especially Lisa, Julie, Katie, Grant, and Brooke.

CONTENTS

PREFACE

> "Human history becomes more and more a race between education and catastrophe."
>
> —H. G. Wells, *The Outline of History*, 1920

To the Student

There is no escaping the importance of mathematics in the modern world. However, for most people, the importance of mathematics lies not in its abstract ideas, but in its application to personal and social issues. This book is designed with such practical considerations in mind. In particular, we've designed this book with three specific purposes:

- To prepare you for the mathematics you will encounter in other **college** courses, particularly core courses in social and natural sciences.
- To develop your ability to reason with quantitative information in a way that will help you achieve success in your **career**.
- To provide you with the critical thinking and quantitative reasoning skills needed to understand major issues in **life**.

We hope this book will be useful to everyone, but it is designed primarily for those who are *not* planning to major in a field that requires advanced mathematical skills. In particular, if you've ever felt any fear or anxiety about mathematics, we've written this book with you in mind. Through this book, you will discover that mathematics is much more important and relevant to your life than you had guessed and not as difficult as previously imagined.

Whatever your interests—social sciences, environmental issues, politics, business and economics, art and music, or any of many other topics—you will find many relevant and up-to-date examples in this book. But the most important idea to take away from this book is that mathematics can help you understand a variety of topics and issues, making you a more aware and better educated citizen. Once you have completed your study of this book, you should be prepared to understand most quantitative issues that you will encounter.

To the Instructor

Whether you've taught this course many times or are teaching it for the first time, you are undoubtedly aware that mathematics courses for nonmajors present challenges that differ from those presented by more traditional courses. First and foremost, there isn't even a clear consensus on what exactly should be taught in these courses. While there's little debate about what mathematical content is necessary for science, technology, engineering, and mathematics (STEM) students—for example, these students all need to learn algebra and calculus—there's great debate about what we should teach non-STEM students, especially the large majority who will *not* make use of formal mathematics in their careers or daily lives.

As a result of this debate, core mathematics courses for non-STEM students fall into a broad and diverse range. Some schools require these students to take a traditional, calculus-track course, such as college algebra. Others have instituted courses that teach students about the ways in which contemporary mathematics contributes to society, focusing on mathematical ideas that students are unlikely to encounter elsewhere. These courses have their merits, and they can certainly be made interesting and relevant, but we believe there are better options because of the following important fact: The vast majority (typically 95%) of non-STEM students will *never* take another college mathematics course after completing their core requirements.

Given this fact, we believe it is essential to teach these students the mathematical ideas that they will *need* for their remaining college course work, their careers, and their daily lives. In other words, while there are many topics that might be new and interesting, we must emphasize those topics that are truly important to the future success of these students. The focus of this approach is less on formal calculation—though some is certainly required—and more on teaching students how to think critically with numerical or mathematical information. In the terminology adopted by MAA, AMATYC, and other mathematical organizations, students need to learn *quantitative reasoning* and to become *quantitatively literate*. There's been a recent rise in the popularity of quantitative reasoning courses for the non-STEM student. This book has been integral to the quantitative reasoning movement for years and continues to be at the forefront as an established entity designed to help you succeed in teaching quantitative reasoning to your students.

The Key to Success: A Context-Driven Approach

Broadly speaking, approaches to teaching mathematics can be divided into two categories:

- A *content-driven* approach is organized by mathematical ideas. After each mathematical topic is presented, examples of its applications are shown.

- A *context-driven* approach is organized by practical contexts. Applications drive the course, and mathematical ideas are presented as needed to support the applications.

The same content can be covered through either approach, but the context-driven approach has an enormous advantage: It motivates students by showing them directly how relevant mathematics is to their lives. In contrast, the content-driven approach tends to come across as "learn this content because it's good for you," causing many students to tune out before reaching the practical applications. For more details, see our article "General Education Mathematics: New Approaches for a New Millennium" (*AMATYC Review,* Fall 1999) or the discussion in the Epilogue of the book *Math for Life* by Jeffrey Bennett (Big Kid Science, 2014).

The Challenge: Winning Over Your Students

Perhaps the greatest challenge in teaching mathematics to students lies in winning them over—that is, convincing them that you have something useful to teach them. This challenge arises because by the time they reach college, many students dislike or fear mathematics. Indeed, the vast majority of students in general education mathematics courses are there not by choice, but because such courses are required for graduation. Reaching your students therefore requires that you teach with enthusiasm and convince them that mathematics is useful and enjoyable.

We've built this book around two important strategies that are designed to help you win students over:

- Confront negative attitudes about mathematics head on, showing students that their fear or loathing is ungrounded and that mathematics actually is relevant to their lives. This strategy is embodied in the Prologue of this book (pages 29–41), which we urge you to emphasize in class. It continues implicitly throughout the rest of the text.
- Focus on goals that are meaningful to students—namely, on the goals of learning mathematics for *college*, *career*, and *life*. Your students will then learn mathematics because they will see how it affects their lives. This strategy forms the backbone of this book, as we have tried to build every unit around topics relevant to college, career, and life.

Modular Structure of the Book

Many of us would love to have a year or more to teach mathematics to general education students. Unfortunately, most schools have only a one-quarter or one-semester mathematics requirement, so we can cover only a fraction of the material we'd cover in an ideal course. This book is therefore organized with a modular structure that allows you to create a course to meet your (or your students') particular interests and constraints. The 12 chapters are organized broadly by contextual areas. Each chapter, in turn, is divided into a set of self-contained *units* that focus on particular concepts or applications. In most cases, you can cover chapters in any order; and while the units within each chapter build sequentially in terms of sophistication, in many cases you can skip certain units, particularly those toward the end of the chapter.

Prerequisite Mathematical Background

Because of its modular structure, this book can be used by students with a wide range of mathematical backgrounds. Many of the units require nothing more than arithmetic and a willingness to think about quantitative issues in new ways. Only a few units use techniques of algebra or geometry, and those skills are reviewed as they arise. This book should therefore be accessible to any student who has completed two or more years of high school mathematics. However, *this book is not remedial*: Although much of the book relies on mathematical techniques from secondary school, the techniques always arise in applications that students generally are not taught in high school and that require students to demonstrate their critical thinking skills.

Changes in the Sixth Edition

We've been pleased by the positive responses of so many users to prior editions of this text. Nevertheless, a book that relies heavily on facts and data always requires a major updating effort to keep it current, and we are always looking for ways to improve clarity and pedagogy. As a result, users of prior editions will find many sections of this book to have been substantially revised or rewritten. Throughout the book we have added more examples and exercises pertaining to vocational careers, which should make the material more relevant to a wider variety of students. We have also made many other changes; while these are too many to list here, they include the following:

Chapter Openers Each chapter now opens with a multiple-choice question designed to illustrate an important way in which the chapter content connects with the book themes of *college*, *careers*, and *life*. These questions can spur lively in-class discussions.

Chapter 1 We significantly revised several units in Chapter 1. In particular, Unit 1A has been expanded to include a focus on evaluation of media information, and we rewrote portions of Units 1C and 1D to help students better understand and interpret Venn diagrams and tests of validity.

Chapter 2 We rewrote and reorganized Units 2A and 2B so basic ideas of units and systems of standardized units are now all covered in Unit 2A while Unit 2B focuses on more sophisticated problem solving with units.

Chapters 3 and 4 These two chapters contain several units that revolve around economic data—such as census data, the consumer price index, interest rates, taxes, and the federal budget—which obviously required major updates given the changes that have occurred in the U.S. economy in the four years since the previous edition was published.

Chapters 5 and 6 These chapters focus on statistical data, which means we updated or replaced large sections of the chapter content to reflect current data.

Chapter 7 We significantly revised the discussion of several key probability ideas to help students better understand them and overcome misconceptions.

Chapters 8 and 9 Units 8B, 8C, and 9C all rely heavily on population data, which means we revised significant portions of these units to reflect the 2010 U.S. Census and updated global demographic data.

Chapter 12 We significantly rewrote major portions of this chapter, particularly in Units 12A and 12C, both to update the political data and to clarify key concepts including those of preference schedules and redistricting.

Pedagogical Features

Besides the main narrative of the text, this book includes the following features, each designed with a specific pedagogical purpose in mind.

Chapter Overview Each chapter begins with a brief overview and a unit-by-unit listing of key content, designed both to show students how the chapter is organized and to help instructors decide which units to cover in class. It is then followed by a multiple-choice question designed to illustrate an important way in which the chapter content connects with the book themes of *college, careers,* and *life*.

Chapter Activity After the overview, each chapter offers an activity designed to spur student discussion of some interesting facet of the topics covered in the chapter. The activities may be done either individually or in small groups.

Time Out to Think Appearing throughout the book, the “Time Out to Think” features pose short conceptual questions designed to help students pause and reflect on important new ideas. They also serve as excellent starting points for class discussions and/or clicker questions.

Summary Boxes Flowing right along with the narrative are boxes that summarize key ideas, definitions, and formulas.

Examples and Case Studies Numbered examples are designed to build understanding and to offer practice with the types of questions that appear in the exercises. Each example is accompanied by a “Now Try” tag that relates the example to specific similar exercises. Occasional case studies go into more depth than the numbered examples.

In Your World These boxes focus on topics that students are likely to encounter in the world around them, whether in the news, in consumer decisions, or in political discussions. Examples include topics such as how to understand jewelry purchases, how to invest money in a sensible way, and how the chained consumer price index (CPI) differs from the standard CPI. This is further enhanced with a section of In Your World exercises in the exercise sets.

Brief Review This feature reviews key mathematical skills that students should have learned previously but in which many students still need review and practice. They appear in the book wherever a particular skill is first needed, and exercises based on the review boxes can be found at the end of the unit.

Using Technology These features give students clear instructions in the use of various technologies for computation, including scientific calculators, Microsoft Excel, and online technologies such as those built in to Google.

Margin Features

- *By the Way* features contain interesting notes and asides relevant to the topic at hand.
- *Historical Note* remarks give historical context to the ideas presented in the chapter.
- *Technical Note* comments contain details that are important mathematically, but generally do not affect students’ understanding of the material.

Mathematical Insight This feature builds upon mathematical ideas in the main narrative but goes somewhat beyond the level of other material in the book. Examples include boxes on the proof of the Pythagorean theorem, on Zeno’s paradox, and on derivations of the financial formulas used for savings plans and mortgage loans.

Chapter Summary Appearing at the end of each chapter, the Chapter Summary offers a detailed outline of the chapter that students can use as a study guide.

Assessment Opportunities

Exercises are presented in various categories, making it easier for instructors to create assignments with a variety of problem types.

Quick Quiz This ten-question quiz appears at the end of each unit and allows students to check whether they understand key concepts before starting the exercise set. Note that students are asked not only to choose the correct multiple-choice answer but also to write a brief explanation of the reasoning behind their choice. Answers are included in the back of the text.

Review Questions Designed primarily for self-study, these questions ask students to summarize the important ideas covered in the unit and generally can be answered simply by reviewing the text.

Does It Make Sense? These qualitative questions test conceptual understanding by asking students to decide whether the given statements are sensible and to explain why or why not.

Basic Skills & Concepts These questions offer practice with the concepts covered in the unit. They can be used for homework assignments or for self-study (answers to most odd-numbered exercises appear in the back of the book). All of these questions are referenced by "Now Try" suggestions in the unit.

Further Applications Through additional applications, these exercises extend the ideas and techniques covered in the text.

In Your World These questions are designed to spur additional research or discussion that will help students relate the unit content to the book themes of college, careers, and life.

Using Technology These exercises, which support the Using Technology features, give students an opportunity to practice calculator or software skills introduced in the text.

Supplements

Instructor Supplements

The following supplements are ONLINE ONLY and are available for download at www.pearsonglobaleditions.com/bennett.

Activity Manual

Shane Goodwin, *Brigham Young University–Idaho*, and Suzanne Topp, *Salt Lake Community College*

- More than 20 activities correlated to the textbook for those who wish to incorporate a more hands-on approach.
- Can be completed by students individually or in a group.
- Includes instructor notes with background information and discussion points.

Instructor's Solutions Manual

James Lapp

- Includes detailed, worked-out solutions to all of the exercises in the text.

Instructor's Testing Manual

Dawn Dabney

- Provides four alternative tests per chapter, including answer keys.

TestGen®

- Enables instructors to build, edit, print, and administer tests, using a computerized bank of questions developed to cover all the objectives of the text.
- Algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button.

PowerPoint® Lecture Presentation

- Classroom presentation slides.
- Includes lecture content and key graphics from the book.

Acknowledgments

A textbook may carry author names, but it is the result of hard work by hundreds of committed individuals. This book has been under development for more than 25 years, and even its beginnings were a group effort, as one of the authors was a member of a committee at the University of Colorado that worked to establish one of the nation's first courses in quantitative reasoning. Since that beginning, the book has benefited from input and feedback from many faculty members and students.

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Prologue

LITERACY FOR THE MODERN WORLD

Equations are just the boring part of mathematics.

—Stephen Hawking, physicist

If you're like most students enrolled in a course using this text, you may think that your interests have relatively little to do with mathematics. But as the quote from Stephen Hawking indicates, mathematics is much more than equations, which is why this text will focus more on mathematical ideas and thinking. As you will see, this type of mathematical thinking is critical today for almost every career, as well as for the decisions and issues that we face daily as citizens in a modern technological society. In this Prologue, we'll discuss why mathematics is so important, why you may be better at it than you think, and how this course can provide you with the quantitative skills needed for your college courses, your career, and your life.

Q

Imagine that you're at a party and you've just struck up a conversation with a dynamic, successful lawyer. Which of the following are you most likely to hear her say during the course of your conversation?

- A "I really don't know how to read very well."
- B "I can't write a grammatically correct sentence."
- C "I'm awful at dealing with people."
- D "I've never been able to think logically."
- E "I'm bad at math."

A

We all know that the answer is E, because we've heard it so many times. Not just from lawyers, but from businessmen and businesswomen, actors and athletes, construction workers and sales clerks, and sometimes even teachers and CEOs. It would be difficult to imagine these same people admitting to any of choices A through D, but many people consider it socially acceptable to say that they are "bad at math." Unfortunately, this social acceptability comes with some very negative social consequences. You can probably think of a few already. For more, see the discussion under Misconception Seven on page 23.

Job Satisfaction

Each chapter in this book will begin with an activity, which you may do individually or in groups. For this Prologue, we begin with an activity that will help you examine the role of mathematics in careers.

Top 20 Jobs for Job Satisfaction

1. Mathematician
2. Actuary (works with insurance statistics)
3. Statistician
4. Biologist
5. Software Engineer
6. Computer Systems Analyst
7. Historian
8. Sociologist
9. Industrial Designer
10. Accountant
11. Economist
12. Philosopher
13. Physicist
14. Parole Officer
15. Meteorologist
16. Medical Laboratory Technician
17. Paralegal Assistant
18. Computer Programmer
19. Motion Picture Editor
20. Astronomer

Source: JobsRated.com.

Everyone wants to find a career path that will bring lifelong job satisfaction, but what careers are most likely to do that? A recent survey evaluated 200 different jobs according to five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. The table to the left shows the top 20 jobs according to this survey. Notice that most of the top 20 jobs require mathematical skills, and all of them require an ability to reason with quantitative information.

You and your classmates can conduct your own smaller study of job satisfaction. There are many ways to do this, but here is one procedure you might try:

1. Each of you should identify at least three people with full-time jobs to interview briefly. You may choose parents, friends, acquaintances, or just someone whose job interests you.
2. Identify an appropriate job category for each interviewee (similar to the categories in the table to the left). Ask each interviewee to rate his or her job on a scale of 1 (worst) to 5 (best) on each of the five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. You can then add the ratings for the five criteria to come up with a total "job satisfaction" rating for each job.
3. Working together as a class, compile the data to rank all the jobs. Show the final results in a table that ranks the jobs in order of job satisfaction.
4. Discuss the results. Are they consistent with the survey results shown in the table? Do they surprise you in any way? Will they have any effect on your own career plans?

What Is Quantitative Reasoning?

Literacy is the ability to read and write, and it comes in varying degrees. Some people can recognize only a few words and write only their names; others read and write in many languages. A primary goal of our educational system is to provide citizens with a level of literacy sufficient to read, write, and reason about the important issues of our time.

Today, the abilities to interpret and reason with **quantitative information**—information that involves mathematical ideas or numbers—are crucial aspects of literacy. These abilities, often called **quantitative reasoning** or **quantitative literacy**, are essential to understanding issues that appear in the news every day. The purpose of this book is to help you gain skills in quantitative reasoning as it applies to issues you will encounter in

- your subsequent coursework,
- your career, and
- your daily life.

Quantitative Reasoning and Culture

Quantitative reasoning enriches the appreciation of both ancient and modern culture. The historical record shows that nearly all cultures devoted substantial energy to mathematics and to science (or to observational studies that predated modern science). Without a sense of how quantitative concepts are used in art, architecture, and science, you cannot fully appreciate the incredible achievements of the Mayans in Central America, the builders of the great city of Zimbabwe in Africa, the ancient Egyptians and Greeks, the early Polynesian sailors, and countless others.

Similarly, quantitative concepts can help you understand and appreciate the works of the great artists. Mathematical concepts play a major role in everything from the work of Renaissance artists like Leonardo da Vinci and Michelangelo to the pop culture of television shows like *The Big Bang Theory*. Other ties between mathematics and the arts can be found in both modern and classical music, as well as in the digital production of music. Indeed, it is hard to find popular works of art, film, or literature that do not rely on mathematics in some way.

Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.

—David Hilbert (1862–1943), mathematician

Quantitative Reasoning in the Work Force

Quantitative reasoning is important in the work force. A lack of quantitative skills puts many of the most challenging and highest-paying jobs out of reach. Table P.1 defines skill levels in language and mathematics on a scale of 1 to 6, and Table P.2 shows the typical levels needed in many jobs.

Note that the occupations requiring high skill levels are generally the most prestigious and highest paying. Note also that most of these occupations call for high skill levels in *both* language and math, refuting the myth that if you're good at language you don't have to be good at mathematics, and vice versa.

TABLE P.1 Skill Levels

Level	Language Skills	Math Skills
1	Recognizes 2500 two- or three-syllable words. Reads at a rate of 95–120 words per minute. Writes and speaks simple sentences.	Adds and subtracts two-digit numbers. Does simple calculations with money, volume, length, and weight.
2	Recognizes 5000–6000 words. Reads 190–215 words per minute. Reads adventure stories and comic books, as well as instructions for assembling model cars. Writes compound and complex sentences with proper grammar and punctuation.	Adds, subtracts, multiplies, and divides all units of measure. Computes ratio, rate, and percentage. Draws and interprets bar graphs.
3	Reads novels and magazines, as well as safety rules and equipment instructions. Writes reports with proper format and punctuation. Speaks well before an audience.	Understands basic geometry and algebra. Calculates discount, interest, profit and loss, markup, and commissions.
4	Reads novels, poems, newspapers, and manuals. Prepares business letters, summaries, and reports. Participates in panel discussions and debates. Speaks extemporaneously on a variety of subjects.	Has true quantitative reasoning abilities. Understands logic, problem solving, ideas of statistics and probability, and modeling.
5	Reads literature, book and play reviews, scientific and technical journals, financial reports, and legal documents. Can write editorials, speeches, and critiques.	Knows calculus and statistics. Is able to deal with econometrics.
6	Same types of skills as level 5, but more advanced.	Works with advanced calculus, modern algebra, and statistics.

Source: Data from the *Wall Street Journal*.

TABLE P. 2 Skill-Level Requirements

Occupation	Language Level	Math Level	Occupation	Language Level	Math Level
Biochemist	6	6	Web page designer	5	4
Computer engineer	6	6	Corporate executive	5	5
Mathematician	6	6	Computer sales agent	4	4
Cardiologist	6	5	Athlete's agent	4	4
Social psychologist	6	5	Management trainee	4	4
Lawyer	6	4	Insurance sales agent	4	4
Tax attorney	6	4	Retail store manager	4	4
Newspaper editor	6	4	Cement mason	3	3
Accountant	5	5	Poultry farmer	3	3
Personnel manager	5	4	Tile setter	3	3
Corporate president	5	5	Travel agent	3	3
Weather forecaster	5	5	Janitor	3	2
Secondary teacher	5	5	Short-order cook	3	2
Elementary teacher	5	4	Assembly-line worker	2	2
Financial analyst	5	5	Toll collector	2	2
Journalist	5	4	Laundry worker	1	1

Source: Data from the *Wall Street Journal.*

Misconceptions about Mathematics

Do you consider yourself to have "math phobia" (fear of mathematics) or "math loathing" (dislike of mathematics)? We hope not—but if you do, you aren't alone. Many adults harbor fear or loathing of mathematics, and unfortunately, these attitudes are often reinforced by classes that present mathematics as an obscure and sterile subject.

In reality, mathematics is not nearly so dry as it sometimes seems in school. Indeed, attitudes toward mathematics often are directed not at what mathematics really is but at some common misconceptions about mathematics. Let's investigate a few of these misconceptions and the reality behind them.

Misconception One: Math Requires a Special Brain

One of the most pervasive misconceptions is that some people just aren't good at mathematics because learning mathematics requires special or rare abilities. The reality is that nearly everyone can do mathematics. All it takes is self-confidence and hard work—the same qualities needed to learn to read, to master a musical instrument, or to become skilled at a sport. Indeed, the belief that mathematics requires special talent found in a few elite people is peculiar to the United States. In other countries, particularly in Europe and Asia, *all* students are expected to become proficient in mathematics.

Of course, different people learn mathematics at different rates and in different ways. For example, some people learn by concentrating on concrete problems, others by thinking visually, and still others by thinking abstractly. No matter what type of thinking style you prefer, you can succeed in mathematics.

We are all mathematicians… [your] forte lies in navigating the complexities of social networks, weighing passions against histories, calculating reactions, and generally managing a system of information that, when all laid out, would boggle a computer.

—A. K. Dewdney, 200% of Nothing

Misconception Two: The Math in Modern Issues Is Too Complex

Some people claim that the advanced mathematical concepts underlying many modern issues are too complex for the average person to understand. It is true that only a few people receive the training needed to work with or discover advanced mathematical concepts. However, most people are capable of understanding enough about the mathematical basis of important issues to develop informed and reasoned opinions.

The situation is similar in other fields. For example, years of study and practice are required to become a proficient professional writer, but most people can read a book. It takes hard work and a law degree to become a lawyer, but most people can understand how the law affects them. And though few have the musical talent of Mozart, anyone can learn to appreciate his music. Mathematics is no different. If you've made it this far in school, you can understand enough mathematics to succeed as an individual and a concerned citizen.

Skills are to mathematics what scales are to music or spelling is to writing. The objective of learning is to write, to play music, or to solve problems—not just to master skills.

—from Everybody Counts, a report of the National Research Council

Misconception Three: Math Makes You Less Sensitive

Some people believe that learning mathematics will somehow make them less sensitive to the romantic and aesthetic aspects of life. In fact, understanding the mathematics that explains the colors of a sunset or the geometric beauty in a work of art can only enhance aesthetic appreciation. Furthermore, many people find beauty and elegance in mathematics itself. It's no accident that people trained in mathematics have made important contributions to art, music, and many other fields.

It is impossible to be a mathematician without being a poet in the soul.

—Sophia Kovalevskaya (1850–1891), Russian mathematician

Misconception Four: Math Makes No Allowance for Creativity

The "turn the crank" nature of the problems in many textbooks may give the impression that mathematics stifles creativity. Some of the facts, formalisms, and skills required for mathematical proficiency are fairly cut and dried, but *using* these mathematical tools takes creativity. Consider designing and building a home. The task demands specific skills to lay the foundation, frame in the structure, install plumbing and wiring, and paint walls. But building the home involves much more: Creativity is needed to develop the architectural design, respond to on-the-spot problems during construction, and factor in constraints based on budgets and building codes. The mathematical skills you've learned in school are like the skills of carpentry or plumbing. Applying mathematics is like the creative process of building a home.

Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand.

—Confucius (c. 551–479 B.C.)

IN YOUR WORLD

People Who Studied Mathematics

The critical thinking skills developed through the study of mathematics are valuable in many careers. The following is only a small sample of people who studied mathematics but became famous for work in other fields. Many of the names come from "Famous Nonmathematicians," a list compiled by Steven G. Buyske, Rutgers University.

Ralph Abernathy, civil rights leader, BS in mathematics, Alabama State University

Corazon Aquino, former president of the Philippines, a mathematics minor

Mayim Bialik, actress on *The Big Bang Theory*, studied mathematics as part of her Ph.D. in neuroscience

Harry Blackmun, former Supreme Court justice, summa cum laude in mathematics, Harvard University

James Cameron, film director, studied physics before leaving college, works in oceanic and space research

Lewis Carroll (Charles Dodgson), author of *Alice in Wonderland,* a mathematician

David Dinkins, former mayor of New York City, BA in mathematics, Howard University

Alberto Fujimori, former president of Peru, MS in mathematics, University of Wisconsin

Art Garfunkel, musician, MA in mathematics, Columbia University

Grace Hopper, computer pioneer and first woman Rear Admiral in the U.S. Navy, Ph.D. in mathematics, Yale University

Mae Jemison, first African-American woman in space, studied mathematics as part of her degree in chemical engineering from Stanford University

John Maynard Keynes, economist, MA in mathematics, Cambridge University

Hedy Lamarr, actress called "the most beautiful woman in Hollywood," invented and patented the mathematical technique of "frequency hopping"

Lee Hsien Loong, politician in Singapore, BA in mathematics, Cambridge University

Brian May, lead guitarist for the band Queen, completed his Ph.D. in astrophysics in 2007, Imperial College

Danica McKellar, actress, BA with highest honors in mathematics, UCLA, and co-discoverer of the Chayes-McKellar-Winn theorem

Edwin Moses, three-time Olympic champion in the 400-meter hurdles, studied mathematics as part of his degree in physics from Morehouse College

Florence Nightingale, pioneer in nursing, studied mathematics and applied it to her work

Natalie Portman, Oscar-winning actress, semifinalist in Intel Science Talent Search and co-author of two published scientific papers

Sally Ride, first American woman in space, studied mathematics as part of her Ph.D. in physics from Stanford University

David Robinson, basketball star, bachelor's degree in mathematics, U.S. Naval Academy

Alexander Solzhenitsyn, Nobel prize–winning Russian author, degrees in mathematics and physics from the University of Rostov

Bram Stoker, author of *Dracula,* studied mathematics at Trinity University, Dublin

Laurence Tribe, Harvard law professor, summa cum laude in mathematics, Harvard University

Virginia Wade, Wimbledon champion, bachelor's degree in mathematics, Sussex University

Misconception Five: Math Provides Exact Answers

A mathematical formula will yield a specific result, and in school that result may be marked right or wrong. But when you use mathematics in real-life situations, answers are never so clear cut. For example:

> *A bank offers simple interest of 3%, paid at the end of one year (that is, after one year the bank pays you 3% of your account balance). If you deposit $1000 today and make no further deposits or withdrawals, how much will you have in your account after one year?*

A straight mathematical calculation seems simple enough: 3% of $1000 is $30; so you should have $1030 at the end of a year. But will you? How will your balance be affected by service charges or taxes on interest earned? What if the bank fails? What if the bank is located in a country in which the currency collapses during the year? Choosing a bank in which to invest your money is a *real* mathematics problem that doesn't necessarily have a simple or definitive solution.

Probably the most harmful misconception is that mathematics is essentially a matter of computation. Believing this is roughly equivalent to believing that writing essays is the same as typing them.

—John Allen Paulos, mathematician

Misconception Six: Math Is Irrelevant to My Life

No matter what your path in college, career, and life, you will find mathematics involved in many ways. A major goal of this text is to show you hundreds of examples in which mathematics applies to everyone's life. We hope you will find that mathematics is not only relevant but also interesting and enjoyable.

Neglect of mathematics works injury to all knowledge…

—Roger Bacon (1214–1294), English philosopher

Misconception Seven: It's OK to Be "Bad at Math"

For our final misconception, let's return to the multiple-choice question in the opening of this Prologue. You'll not only hear many otherwise intelligent people say "I'm bad at math," but it's sometimes said almost as a point of pride, with no hint of embarrassment. Yet the statement often isn't even true. Our successful lawyer, for example, almost certainly did well in all subjects in school, including math, so she is more likely expressing an attitude than a reality.

Unfortunately, this type of attitude can cause a lot of damage. Mathematics underlies nearly everything in modern society, from the daily financial decisions that all of us must make to the way in which we understand and approach global issues of the economy, politics, and science. We cannot possibly hope to act wisely if we approach mathematical ideas with a poor attitude. Moreover, it's an attitude that can easily spread to others. After all, if a child hears a respected adult saying that he or she is "bad at math," the child may be less inspired to do well.

So before you begin your coursework, think about your own attitudes toward mathematics. There's no reason why anyone should be "bad at math" and every reason to develop skills of mathematical thinking. With a good attitude and some hard work, by the end of your course you'll not only be better at math, but you'll be helping future generations by making it socially unacceptable for anyone to be "bad at math."

You must be the change you wish to see in the world.

—Mahatma Gandhi (1869–1948)

What *Is* Mathematics?

In discussing misconceptions, we identified what mathematics is *not*. Now let's look at what mathematics *is*. The word *mathematics* is derived from the Greek word *mathematikos,* which means "inclined to learn." Literally speaking, to be mathematical is to be curious, open-minded, and interested in always learning more! Today, we tend to look at mathematics in three different ways: as the sum of its branches, as a way to model the world, and as a language.

Mathematics as the Sum of Its Branches

As you progressed through school, you probably learned to associate mathematics with some of its branches. Among the better known branches of mathematics are these:

- **logic**—the study of principles of reasoning;
- **arithmetic**—methods for operating on numbers;
- **algebra**—methods for working with unknown quantities;
- **geometry**—the study of size and shape;
- **trigonometry**—the study of triangles and their uses;
- **probability**—the study of chance;
- **statistics**—methods for analyzing data; and
- **calculus**—the study of quantities that change.

One can view mathematics as the sum of its branches, but in this book we'll focus on how different branches of mathematics support the more general goals of quantitative thinking and critical reasoning.

Mathematics as a Way to Model the World

Mathematics also may be viewed as a tool for creating models, or representations of real phenomena. Modeling is not unique to mathematics. For example, a road map is a model that represents the roads in some region.

Mathematical models can be as simple as a single equation that predicts how the money in your bank account will grow or as complex as a set of thousands of interrelated equations and parameters used to represent the global climate. By studying models, we gain insight into otherwise unmanageable problems. A global climate model, for example, can help us understand weather systems and ask "what if" questions about how human activity may affect the climate. When a model is used to make a prediction that does *not* come true, it points out areas where further research is needed. Today, mathematical modeling is used in nearly every field of study. Figure P.1 indicates some of the many disciplines that use mathematical modeling to solve problems.

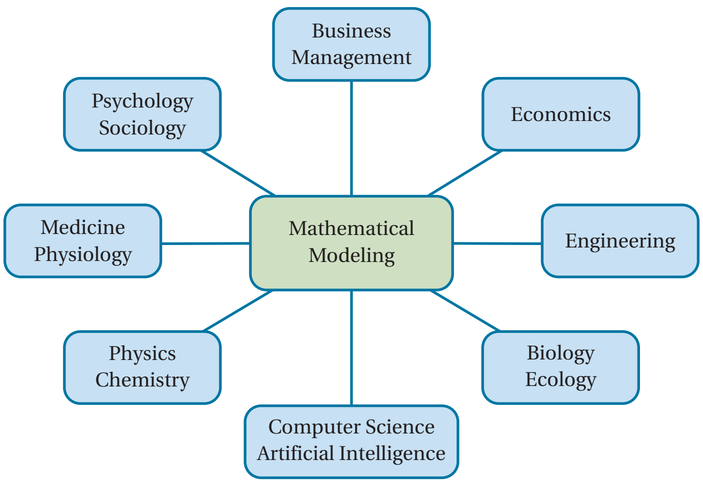


FIGURE P.1

Mathematics as a Language

A third way to look at mathematics is as a language with its own vocabulary and grammar. Indeed, mathematics often is called "the language of nature" because it is so useful for modeling the natural world. As in any language, different degrees of fluency are possible. From this point of view, quantitative literacy is the level of fluency required for success in today's world.

The idea of mathematics as a language also is useful in thinking about how to *learn* mathematics. Table P.3 compares learning mathematics to learning a language and learning art.

The Book of Nature is written in the language of mathematics.

—Galileo

TABLE P.3 Learning Mathematics: An Analogy to Language and Art

Learning a Language	Learning the Language of Art	Learning the Language of Mathematics
Learn many styles of speaking and writing, such as essays, poetry, and drama.	Learn many styles of art, such as classical, renaissance, impressionist, and modern.	Learn techniques from many branches of mathematics, such as arithmetic, algebra, and geometry.
Place literature in context through the history and social conditions under which it was created.	Place art in context through the history and social conditions under which it was created.	Place mathematics in context through its history, purposes, and applications.
Learn the elements of language—such as words, parts of speech (nouns, verbs, etc.), and rules of grammar—and practice their proper use.	Learn the elements of visual form— such as lines, shapes, colors, and textures—and practice using them in your own art work.	Learn the elements of mathematics—such as numbers, variables, and operations—and practice using them to solve simple problems.
Critically analyze language in forms such as novels, short stories, essays, poems, speeches, and debates.	Critically analyze works of art including painting, sculpture, architecture, and photography.	Critically analyze quantitative information in mathematical models, statistical studies, economic forecasts, investment strategies, and more.
Use language creatively for your own purposes, such as writing a term paper or story or engaging in debate.	Use your sense of art creatively, such as in designing your house, taking a photograph, or making a sculpture.	Use mathematics creatively to solve problems you encounter and to help you understand issues in the modern world.

How to Succeed in Mathematics

If you are reading this book, you probably are enrolled in a mathematics course. The keys to success in your course include approaching the material with an open and optimistic frame of mind, paying close attention to how useful and enjoyable mathematics can be in *your* life, and studying effectively and efficiently. The following sections offer a few specific hints that may be of use as you study.

Using This Book

Before we get into more general strategies for studying, here are a few guidelines that will help you use *this* book most effectively.

- Before doing any assigned exercises, read assigned material *twice*:
  - On the first pass, read quickly to gain a "feel" for the material and concepts presented.
  - On the second pass, read the material in more depth and work through the examples carefully.

- During the second reading, take notes that will help you when you go back to study later. In particular:
  - Use the margins! The wide margins in this textbook are designed to give you plenty of room to make notes as you study.
  - Don't highlight—underline! Using a pen or pencil to underline material requires greater care than highlighting and therefore helps to keep you alert as you study.
- After you complete the reading, and again when studying for exams, make sure you can answer the Quick Quiz and Review Questions at the end of each unit.
- You'll learn best by *doing*, so do plenty of the end-of-unit exercises. In particular, try some of the exercises that have answers in the back of the book, in addition to those assigned by your instructor.

Budgeting Your Time

The single most important key to success in any college course is to spend enough time studying. A general rule of thumb for college classes is that you should expect to study about 2 to 3 hours per week *outside* class for each unit of credit. For example, a student taking 15 credit hours should spend 30 to 45 hours each week studying outside of class. Combined with time in class, this works out to a total of 45 to 60 hours per week—not much more than the time required of a typical job, and you get to choose your own hours. Of course, if you are working or taking care of a family while you attend school, you will need to budget your time carefully.

The following table gives some rough guidelines for how you might divide your studying time in your mathematics course. If you are spending fewer hours than these guidelines suggest, you could probably improve your grade by studying more. If you are spending more hours than these guidelines suggest, you may be studying inefficiently; in that case, you should talk to your instructor about how to study more effectively.

If Your Course Is	Time for Reading the Assigned Text (per Week)	Time for Homework Assignments (per Week)	Time for Review and Test Preparation (Average per Week)	Total Study Time (per Week)
3 credits	1 to 2 hours	3 to 5 hours	2 hours	6 to 9 hours
4 credits	2 to 3 hours	3 to 6 hours	3 hours	8 to 12 hours
5 credits	2 to 4 hours	4 to 7 hours	4 hours	10 to 15 hours

General Strategies for Studying

- Budget your time effectively. One or two hours each day is more effective, and far less painful, than studying all night before homework is due or before exams.
- Engage your brain. Learning is an active process, not a passive experience. Whether you are reading, listening to a lecture, or working on assignments, always make sure that your mind is actively engaged. If you find your mind drifting or falling asleep, make a conscious effort to revive yourself or take a break if necessary.

- Don't miss class. Listening to lectures and participating in class activities and discussions are much more effective than reading someone else's notes. Active participation will help you retain what you are learning.
- Be sure to complete any assigned reading *before* the class in which it will be discussed. This is crucial because class lectures and discussions are designed to help reinforce key ideas from the reading.
- Start your homework early. The more time you allow yourself, the easier it is to get help if you need it. If a concept gives you trouble, first try additional reading or studying beyond what has been assigned. If you still have trouble, ask for help: You surely can find friends, peers, or teachers who will help you learn.
- Working together with friends can be valuable in helping you understand difficult concepts. However, be sure that you learn *with* your friends and do not become dependent on them.
- Don't try to multitask. A large body of research shows that human beings simply are not good at multitasking: When we attempt it, we do more poorly at all of the individual tasks. And in case you think you are an exception, the same research found that those people who believed they were best at multitasking were actually the worst! When it is time to study, turn off your electronic devices, find a quiet spot, and give your work a focused effort of concentration.

Preparing for Exams

- Rework exercises and other assignments. Try additional exercises to be sure you understand the concepts. Study your assignments, quizzes, and exams from earlier in the semester.
- Study your notes from lectures and discussions, and reread relevant sections in your textbook. Pay attention to what your instructor expects you to know for an exam.
- Study individually *before* joining a study group with friends. Study groups are effective only if *every* individual comes prepared to contribute.
- Don't stay up too late before an exam. Don't eat a big meal within an hour of the exam (thinking is more difficult when blood is going to the digestive system).
- Try to relax before and during the exam. If you have studied effectively, you are capable of doing well. Staying relaxed will help you think clearly.

Presenting Homework and Writing Assignments

All work that you turn in should be of *collegiate* quality: neat and easy to read, well organized, and demonstrating mastery of the subject matter. Future employers and teachers will expect this quality of work. Moreover, although submitting homework of collegiate quality requires "extra" effort, it serves two important purposes directly related to learning:

1. The effort you expend in clearly explaining your work solidifies your understanding. In particular, research has shown that writing and speaking trigger different areas of your brain. By writing something down—even when you think you already understand it—your learning is reinforced by involving other areas of your brain.
2. By making your work clear and self-contained (that is, making it a document that you can read without referring to the questions in the text), you will have a much more useful study guide when you review for a quiz or exam.

The following guidelines will help ensure that your assignments meet the standards of collegiate quality:

- Always use proper grammar, proper sentence and paragraph structure, and proper spelling. Do not use texting shorthand.
- All answers and other writing should be fully self-contained. A good test is to imagine that a friend is reading your work and to ask yourself whether the friend would understand exactly what you are trying to say. It is also helpful to read your work out loud to yourself, making sure that it sounds clear and coherent.
- In problems that require calculation:
  - Be sure to *show your work* clearly. By doing so, both you and your instructor can follow the process you used to obtain an answer. Also, please use standard mathematical symbols, rather than "calculator-ese." For example, show multiplication with the × symbol (not with an asterisk), and write 10^5, not 10^5 or 10E5.
  - *Word problems should have word answers*. That is, after you have completed any necessary calculations, any problem stated in words should be answered with one or more *complete sentences* that describe the point of the problem and the meaning of your solution.
  - Express your word answers in a way that would be *meaningful* to most people. For example, most people would find it more meaningful if you express a result of 720 hours as 1 month. Similarly, if a precise calculation yields an answer of 9,745,600 years, it may be more meaningful in words as "nearly 10 million years."
- Include illustrations whenever they help explain your answer, and make sure your illustrations are neat and clear. For example, if you graph by hand, use a ruler to make straight lines. If you use software to make illustrations, be careful not to make them overly cluttered with unnecessary features.
- If you study with friends, be sure that you turn in your own work stated in your own words—you should avoid anything that might even give the *appearance* of possible academic dishonesty.

Prologue

DISCUSSION QUESTIONS

1. **Mathematics in Modern Issues.** Describe at least one way that mathematics is involved in each issue below.

   Example: The spread of AIDS: Mathematics is used to study the probability of contracting AIDS.

   a. The long-term viability of the Social Security system
   b. The appropriate level for the federal gasoline tax
   c. National health care policy
   d. Job discrimination against women or ethnic minorities
   e. Effects of population growth (or decline) on your community
   f. Possible bias in standardized tests (e.g., the SAT)
   g. The degree of risk posed by carbon dioxide emissions
   h. Immigration policy of the United States
   i. Violence in public schools
   j. Whether certain types of guns or ammunition should be banned
   k. An issue of your choice from today's news

2. **Quantitative Concepts in the News.** Identify the major unresolved issue discussed in today's news. List at least three areas in which quantitative concepts play a role in the policy considerations of this issue.

3. **Mathematics and the Arts.** Choose a well-known historical figure in a field of art in which you have a personal interest (e.g., a painter, sculptor, musician, or architect). Briefly describe how mathematics played a role in or influenced that person's work.

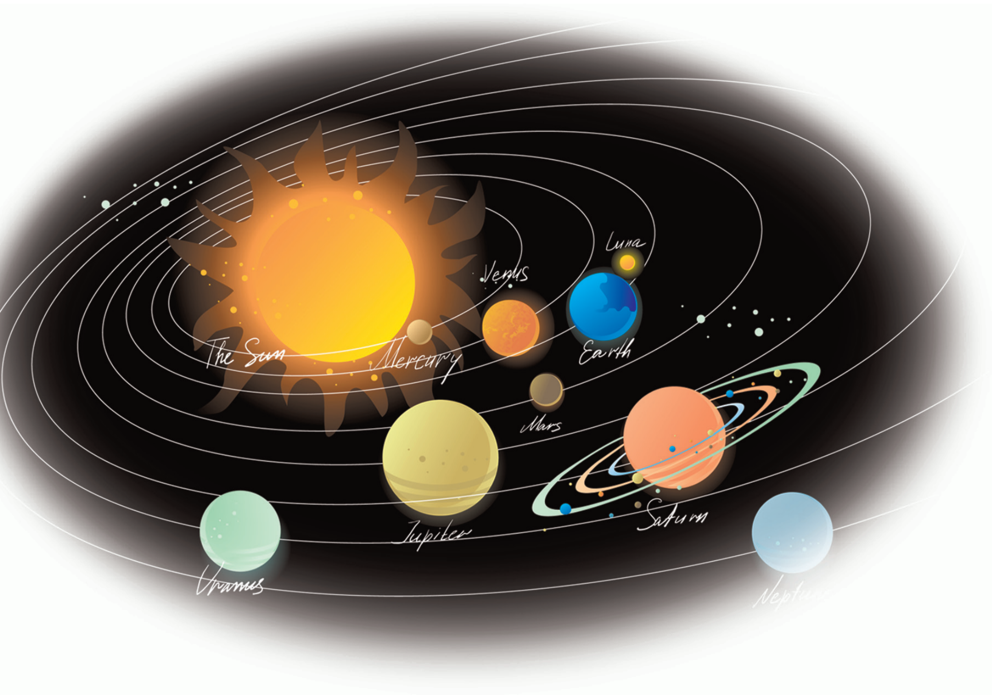
4. **Quantitative Literature.** Choose a favorite work of literature (poem, play, short story, or novel). Describe one or more instances in which quantitative reasoning is helpful in understanding the subtleties intended by the author.

5. **Your Quantitative Major.** Identify ways in which quantitative reasoning is important within your major field of study. (If you haven't yet chosen a major, pick a field that you are considering for your major.)
6. **Career Preparation.** Realizing that most Americans change careers several times during their lives, identify at least three occupations in Table 2 that interest you. Do you have the necessary skills for them at this time? If not, how can you acquire these skills?
7. **Attitudes Toward Mathematics.** What is your attitude toward mathematics? If you have a negative attitude, can you identify when that attitude developed? If you have a positive attitude, can you explain why? How might you encourage someone with a negative attitude to become more positive?
8. **"Bad at Math" as a Social Disease.** Discuss reasons why many people think being "bad at math" is socially acceptable and how we as a society can change those attitudes. If you were a teacher, what would you do to ensure that your students develop positive attitudes toward mathematics?

1 THINKING CRITICALLY

The primary goal of this text is to help you develop the quantitative reasoning skills you will need to succeed in other college courses, in your career, and in your life as a citizen in an increasingly complex world. Quantitative reasoning combines basic mathematical skills—most of which you already have—with the ability to approach problems in a critical and analytical way. For this reason, we devote this first chapter to studying ideas of logic that will develop your ability to think critically.

Q **Perhaps you, like millions of others, have received this message: "On August 27, Mars will look as large and bright as the full Moon. Don't miss it, because no one alive today will ever see this again." This claim:**



- **A** is true, because on this date Mars will be closer to Earth than any time in thousands of years.
- **B** is true, because on this date Mars will be closer to Earth than the Moon.
- **C** was true for the year 2012, but not for other years.
- **D** is false.
- **E** is partially true: Mars really will be this bright, but it happens every year on August 27, so you'll see it again.

> **Mathematics is just logic with numbers attached.**
>
> —Marilyn vos Savant, American author

If you're like most students, you may be wondering what this question has to do with math. The answer is "a lot." To begin with, logic is actually a branch of mathematics, and you can use logic to analyze the claim about Mars. Beyond that, the question also involves mathematics on several deeper levels. For example, the statement "Mars will look as large… as the full Moon" is a statement about *angular size*, which is a mathematical way of expressing how large an object appears to your eye. In addition, a full understanding of the claim requires understanding how the Moon orbits Earth and planets orbit the Sun, which means understanding that orbits have the mathematical shape called an *ellipse* and obey precise mathematical laws.

So what's the answer? Here's a key hint: Think about the fact that Mars is a planet orbiting the Sun while the Moon orbits Earth. Given that fact, ask yourself when, if ever, Mars could appear as large and bright as the full Moon. To see the answer and discussion, go to Example 11 on Page 38.

UNIT 1A

Living in the Media Age: Explore common fallacies, or deceptive arguments, and learn how to avoid them.

UNIT 1B

Propositions and Truth Values: Study basic components of logic, including propositions, truth values, truth tables, and the logical connectors *and, or,* and *if…then.*

UNIT 1C

Sets and Venn Diagrams: Understand sets, and use Venn diagrams to visualize relationships among sets.

UNIT 1D

Analyzing Arguments: Learn to distinguish and evaluate basic inductive and deductive arguments.

UNIT 1E

Critical Thinking in Everyday Life: Apply logic to common situations in everyday life.

ACTIVITY

Bursting Bubble

Use this activity to gain a sense of the kinds of problems this chapter will enable you to study.

The global economy is still recovering from the deep recession and financial crisis that began in 2007 and led to massive bank bailouts, huge increases in unemployment, and many other severe economic consequences. While the recession had many causes, the clear trigger that set it off was a fairly sudden collapse in housing prices. This collapse led many homeowners to default on their home mortgages, which in turn created a crisis for banks and other institutions that bought, sold, or insured home mortgages. If we hope to avoid similar crises in the future, a key question is whether there were early warning signs that might have allowed both individuals and policy makers to make decisions that could have prevented the problems before they occurred.

Figure 1.A shows how average (median) home prices have compared to average income over the past several decades. A ratio of 3.0, for example, means that the average home price is three times the average annual household income of Americans; that is, if you had a household income of $50,000 per year and bought an average house, the price of your house would be $150,000. Notice that the ratio remained below about 3.5 until 2001, when it suddenly started shooting up, which is why the period from 2001 to about 2006 is said to have been marked by a *housing bubble*.

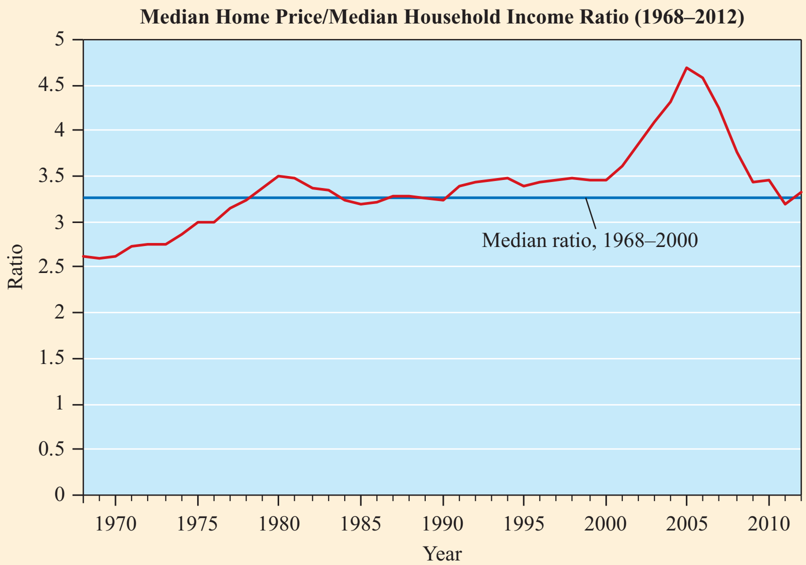


FIGURE 1.A *Source:* Data from *The State of the Nation's Housing 2013*, used with permission from the Joint Center for Housing Studies of Harvard University. All rights reserved.

Was the change in the home price to income ratio a warning sign that should have been heeded? Use your powers of logic—the topic of this chapter—to discuss the following questions.

1. Consider a family with an annual income of $50,000. If they bought an average home, how much would they have spent in 2000, when the home price to income ratio was about 3.5? How much would they have spent in 2005, when the ratio was about 4.7?

2. In percentage terms, a rise in the ratio from 3.5 to 4.7 is an increase of nearly 35%. Because the ratio was *below* 3.5 for decades before 2001, we can conclude that the average home

was at least 35% more expensive relative to income in 2005 than it had been historically. What can you infer about how the percentage of income that a family spent on housing changed during the housing bubble?

3. In general, a family can increase the percentage of its income that it spends on housing only if some combination of the following three things happens: (1) its income increases, so it can afford to spend more of it on housing; (2) it cuts expenses in other areas; or (3) it borrows more money. Based on your understanding of the housing crisis, what happened in most cases during the housing bubble?
4. Overall, do you think it was inevitable that the bubble would burst? Why or why not?
5. How could you use the data on the home price to income ratio to help *you* make a decision about how much to spend when you are looking to buy a home?
6. Bonus: As home prices rose during the bubble, the optimists claimed that the higher prices could be sustained. Do a bit of Web research to learn how they justified this belief. Do you think their arguments sounded reasonable at the time? Do they still sound reasonable with hindsight?
7. Additional Research: The data shown here reflect a nationwide average, but the home price to income ratio varies considerably in different cities and regions. Find data for a few different cities or regions, and discuss the differences.

UNIT 1A Living in the Media Age

We are living in what is sometimes called the "media age," because we are in almost constant contact with media of some sort. Some of the media content is printed in books, newspapers, magazines, and billboards. Much more is delivered electronically through the Internet, tablets and smart phones, television, movies, and more. Most people rely on these media sources for information, which means they form opinions and beliefs based on these same sources.

Unfortunately, much of the information in the media is either inaccurate or biased, designed less to inform us than to convince us of something that may or may not be true. As a result, the only way to make sense of the media information bombardment is to equip yourself with an understanding of the ways in which people try to manipulate your views. In this first unit, we'll explore a few of the tools that can help you navigate the media intelligently. These tools will also provide a foundation for the critical thinking and quantitative reasoning that we'll focus on in the rest of this book.

The Concept of Logical Argument

If you read the comments that follow many news articles on the Web, you'll often see heated discussions that might look much like this "argument" between two classmates.

Ethan: *The death penalty is immoral.*
Jessica: *No it isn't.*
Ethan: *Yes it is! Judges who give the death penalty should be impeached.*
Jessica: *You don't even know how the death penalty is decided.*

People generally quarrel because they cannot argue.

—G. K. Chesterton (1874–1936), English author

Ethan: *I know a lot more than you know!*

Jessica: *I can't talk to you; you're an idiot!*

This type of argument may be common, but it accomplishes little. It doesn't give either person insight into the other's thinking, and it is unlikely to change either person's opinion. Fortunately, there is a better way to argue. We can use skills of **logic**—the study of the methods and principles of reasoning. Arguing logically may still not change either person's position, but it can help them understand each other.

In logic, the term **argument** refers to a reasoned or thoughtful process. Specifically, an argument uses a set of facts or assumptions, called **premises**, to support a **conclusion.** Some arguments provide strong support for their conclusions, but others do not. An argument that fails to make a compelling case for its conclusion may contain some error in reasoning, or **fallacy** (from the Latin for "deceit" or "trick"). In other words, a fallacious argument tries to persuade in a way that doesn't really make sense when analyzed carefully.

Definitions

Logic is the study of the methods and principles of reasoning.

An **argument** uses a set of facts or assumptions, called **premises**, to support a **conclusion.**

A **fallacy** is a deceptive argument—an argument in which the conclusion is not well supported by the premises.

BY THE WAY

Advertisements are filled with fallacies, largely because there's usually no really good reason why you should buy some particular brand or product. Still, they must work, because U.S. businesses spend almost \$200 billion per year—or nearly \$700 per person in the United States—trying to get you to buy stuff.

Common Fallacies

Fallacies in the media are so common that it is nearly impossible to avoid them. Moreover, fallacies often sound persuasive, despite their logical errors, in part because public relations specialists have spent billions of dollars researching how to persuade us to buy products, vote for candidates, or support particular policies. Because fallacies are so common, it is important to be able to recognize them. We therefore begin our study of critical thinking with examples of a few of the most common fallacies. The fallacy in each example has a fancy name, but learning the names is far less important than learning to recognize the faulty reasoning. The experience you gain by analyzing fallacies will provide a foundation upon which to build additional critical thinking skills.

EXAMPLE 1 Appeal to Popularity

"Ford makes the best pickup trucks in the world. More people drive Ford pickups than any other light truck."

Analysis The first step in dealing with any argument is recognizing which statements are premises and which are conclusions. This argument tries to make the case that *Ford makes the best pickup trucks in the world,* so this statement is its conclusion. The only evidence it offers to support this conclusion is the statement *more people drive Ford pickups than any other light truck.* This is the argument's only premise. Overall, this argument has the form

Premise: More people drive Ford pickups than any other light truck.

Conclusion: Ford makes the best pickup trucks in the world.

Note that the original written argument states the conclusion before the premise. Such "backward" structures are common in everyday speech and are perfectly legitimate as long as the argument is well reasoned. In this case, however, the reasoning is faulty.

The fact that more people drive Ford pickups does not necessarily mean that they are the best trucks.

This argument suffers from the fallacy of *appeal to popularity* (or *appeal to majority*), in which the fact that large numbers of people believe or act some way is used inappropriately as evidence that the belief or action is correct. We can represent the general form of this fallacy with a diagram in which the letter *p* stands for a particular statement (Figure 1.1). In this case, *p* stands for the statement *Ford makes the best pickup trucks in the world.* ▶ **Now try Exercise 11.**

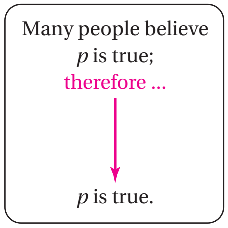


FIGURE 1.1 The fallacy of appeal to popularity. The letters *p* and *q* (used in later diagrams) represent statements.

EXAMPLE 2 False Cause

"I placed the quartz crystal on my forehead, and in five minutes my headache was gone. The crystal made my headache go away."

Analysis We identify the premises and conclusion of this argument as follows:

Premise: I placed the quartz crystal on my forehead.

Premise: Five minutes later my headache was gone.

Conclusion: The crystal made my headache go away.

The premises tell us that one thing (crystal on forehead) happened before another (headache went away), but they don't prove any connection between them. That is, we cannot conclude that the crystal *caused* the headache to go away.

This argument suffers from the fallacy of *false cause*, in which the fact that one event came before another is incorrectly taken as evidence that the first event *caused* the second event. We can represent this fallacy with a diagram in which *A* and *B* represent two different events (Figure 1.2). In this case, *A* is the event of putting the crystal on the forehead and *B* is the event of the headache going away. (We'll discuss how cause *can* be established in Chapter 5.) ▶ **Now try Exercise 12.**

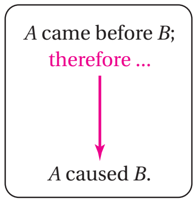


FIGURE 1.2 The fallacy of false cause. The letters *A* and *B* represent events.

EXAMPLE 3 Appeal to Ignorance

"Scientists have not found any concrete evidence of aliens visiting Earth. Therefore, anyone who claims to have seen a UFO must be hallucinating."

Analysis If we strip the argument to its core, it says this:

Premise: There's no proof that aliens have visited Earth.

Conclusion: Aliens have not visited Earth.

The fallacy should be clear: A lack of proof of alien visits does not mean that visits have not occurred. This fallacy is called *appeal to ignorance* because it uses ignorance (lack of knowledge) about the truth of a proposition to conclude the opposite (Figure 1.3). We sometimes sum up this fallacy with the statement "*An absence of evidence is not evidence of absence.*" ▶ **Now try Exercise 13.**

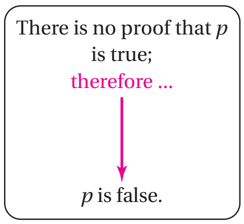


FIGURE 1.3 The fallacy of appeal to ignorance.

Time Out to Think Suppose a person is tried for a crime and found *not* guilty. Can you conclude that the person is innocent? Why or why not? Why do you think our legal system demands that prosecutors prove guilt, rather than demanding that defendants (suspects) prove innocence? How is this idea related to the fallacy of appeal to ignorance?

EXAMPLE 4 Hasty Generalization

"Two cases of childhood leukemia have occurred along the street where the high-voltage power lines run. The power lines must be the cause of these illnesses."

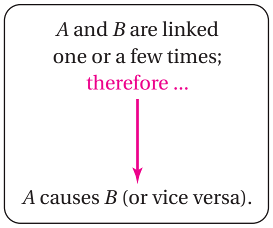


FIGURE 1.4 The fallacy of hasty generalization.

Analysis The premise of this argument cites two cases of leukemia, but two cases are not enough to establish a pattern, let alone to conclude that the power lines caused the illnesses.

The fallacy here is *hasty generalization,* in which a conclusion is drawn from an inadequate number of cases or cases that have not been sufficiently analyzed. If any connection between power lines and leukemia exists, it would have to be established with far more evidence than is provided in this argument. (In fact, decades of research have found no connection between power lines and illness.) We can represent this fallacy with a diagram in which *A* and *B* represent two linked events (Figure 1.4).

▶ **Now try Exercise 14.**

EXAMPLE 5 Limited Choice

"You don't support the President, so you are not a patriotic American."

Analysis This argument has the form

Premise: You don't support the President.
Conclusion: You are not a patriotic American.

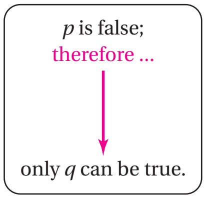


FIGURE 1.5 The fallacy of limited choice.

The argument suggests that there are only two types of Americans: patriotic ones who support the President and unpatriotic ones who don't. But there are many other possibilities, such as being patriotic while disliking a particular President.

This fallacy is called *limited choice* (or *false dilemma*) because it artificially precludes choices that ought to be considered. Figure 1.5 shows one common form of this fallacy. Limited choice also arises with questions such as "Have you stopped smoking?" Because both *yes* and *no* answers imply that you smoked in the past, the question precludes the possibility that you never smoked. (In legal proceedings, questions of this type are disallowed because they attempt to "lead the witness.") Another simple and common form of this fallacy is "You're wrong, so I must be right."

▶ **Now try Exercise 15.**

EXAMPLE 6 Appeal to Emotion

In ads for Michelin tires, a picture of a baby is shown with the words "because so much is riding on your tires."

Analysis If we can consider this an argument at all, it has the form

Premise: You love your baby.
Conclusion: You should buy Michelin tires.

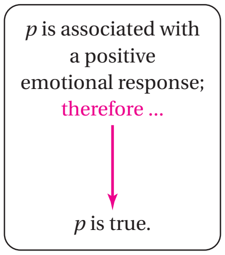


FIGURE 1.6 The fallacy of appeal to emotion.

The advertisers hope that the love you feel for a baby will make you want to buy their tires. This attempt to evoke an emotional response as a tool of persuasion represents the fallacy of *appeal to emotion.* Figure 1.6 shows its form when the emotional response is positive. Sometimes the appeal is to negative emotions. For example, the statement *if my opponent is elected, your tax burden will rise* tries to convince you that electing the other candidate will lead to consequences you won't like. (In this negative form, the fallacy is sometimes called *appeal to force.*)

▶ **Now try Exercise 16.**

EXAMPLE 7 Personal Attack

Gwen: You should stop drinking because it's hurting your grades, endangering people when you drink and drive, and destroying your relationship with your family.

Merle: I've seen you drink a few too many on occasion yourself!

Analysis Gwen's argument is well reasoned, with premises offering strong support for her conclusion that Merle should stop drinking. Merle rejects this argument by noting that Gwen sometimes drinks too much herself. Even if Merle's claim is true, it is irrelevant to Gwen's point. Merle has resorted to attacking Gwen personally rather than arguing logically, so we call this fallacy *personal attack* (Figure 1.7). (It is also called *ad hominem,* Latin for "to the person.")

The fallacy of personal attack can also apply to groups. For example, someone might say, "This new bill will be an environmental disaster because its sponsors received large campaign contributions from oil companies." This argument is fallacious because it doesn't challenge the provisions of the bill, but only questions the motives of the sponsors. ▶ **Now try Exercise 17.**

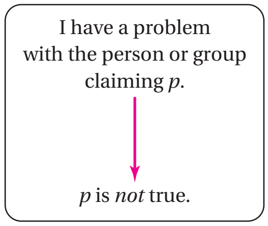


FIGURE 1.7 The fallacy of personal attack.

Time Out to Think A person's (or group's) character, circumstances, and motives occasionally *are* logically relevant to an argument. That is why, for example, witnesses in criminal cases often are asked questions about their personal lives. If you were a judge, how would you decide when to allow such questions?

EXAMPLE 8 Circular Reasoning

"Society has an obligation to provide health insurance because health care is a right of citizenship."

Analysis This argument states the conclusion (*society has an obligation to provide health insurance*) before the premise (*health care is a human right*). But if you read carefully, you'll recognize that the premise and the conclusion both say essentially the same thing, as social obligations are generally based on definitions of accepted rights. This argument therefore suffers from *circular reasoning* (Figure 1.8). ▶ **Now try Exercise 18.**

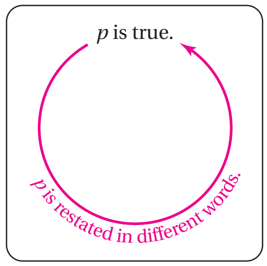


FIGURE 1.8 The fallacy of circular reasoning.

EXAMPLE 9 Diversion (Red Herring)

"We should not continue to fund cloning research because there are so many ethical issues involved. Decisions are based on ethics, and we cannot afford to have too many ethical loose ends."

Analysis The argument begins with its conclusion—*we should not continue to fund cloning research.* However, the discussion is all about ethics. This argument represents the fallacy of *diversion* (Figure 1.9) because it attempts to divert attention from the real issue (funding for cloning research) by focusing on another issue (ethics). The issue to which attention is diverted is sometimes called a *red herring.* (A herring is a fish that turns red when rotten. Use of the term *red herring* to mean a diversion can be traced back to the 19th century, when British fugitives discovered that they could divert bloodhounds from their pursuit by rubbing a red herring across their trail.) Note that personal attacks (see Example 7) are often used as diversions. ▶ **Now try Exercise 19.**

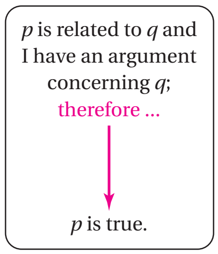


FIGURE 1.9 The fallacy of diversion.

EXAMPLE 10 Straw Man

Suppose that the mayor of a large city proposes decriminalizing drug possession in order to reduce overcrowding in jails and save money on enforcement. His challenger in the upcoming election says, "The mayor doesn't think there's anything wrong with drug use, but I do."

Analysis The mayor did not say that drug use is acceptable. His proposal for decriminalization is designed to solve another problem—overcrowding of jails—and tells us nothing about his general views on drug use. The speaker has distorted the mayor's views. Any argument based on a distortion of someone's words or beliefs is called a *straw man* (Figure 1.10). The term comes from the idea that the speaker has used a poor representation of a person's beliefs in the same way that a straw man is a poor representation of a real man. A straw man is similar to a diversion. The primary difference is that a diversion argues against an unrelated issue, while the straw man argues against a distorted version of the real issue. ▶ Now try Exercise 20.

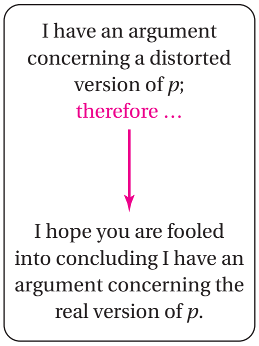


FIGURE 1.10 The straw man fallacy.

Evaluating Media Information

The fallacies we've discussed represent only a small sample of the many tactics used by individuals, groups, and companies seeking to shape your opinions. There are no foolproof ways to be sure that a particular piece of media information is reliable. However, there are a few guidelines, summarized in the following box, that can be helpful. Keep these ideas in mind not only as you evaluate media information, but also as you continue your study in this course. As you will see, much of the rest of this book is devoted to learning to evaluate *quantitative* information using the same general criteria given in the box.

Five Steps to Evaluating Media Information

1. **Consider the source.** Is the source of the information clear? Does the source have credibility on this issue?
2. **Check the date**. Can you determine when the information was written? Is it still relevant, or is it outdated?
3. **Validate accuracy**. Can you validate the information from other sources? Do you have good reason to believe it is accurate? Does it contain anything that makes you suspicious?
4. **Watch for hidden agendas**. Is the information presented fairly and objectively, or is it manipulated to serve some particular or hidden agenda?
5. **Don't miss the big picture**. Even if a piece of media information passes all the above tests, step back and consider whether it makes sense. For example, does it conflict with other things you think are true, and if so, how can you resolve the conflict?

BY THE WAY

The Mars claim has been circulated so much that it is known as the "Mars hoax." While it is untrue, Mars *does* become as bright or brighter than any star in our night sky for several weeks around the times when it comes closest to us in its orbit, which happens about every 26 months. Recent or upcoming dates when Mars reaches peak brightness are: April 8, 2014; May 22, 2016; and July 27, 2018.

EXAMPLE 11 Mars in the Night Sky

Evaluate the Mars claim from the chapter opener: "On August 27, Mars will look as large and bright as the full Moon. Don't miss it, because no one alive today will ever see this again."

Analysis Let's apply the five steps for evaluating media information:

1. **Consider the source.** The original source of the claim is not given, which means you have no way to know if the source is authoritative. This should make you suspicious of the claim.
2. **Check the date.** Although the claim sounds specific in citing August 27 for the event, no year is given, so you have no way to know if the claim is intended to apply to this year, every year, or a particular past or future year. This should increase your concern about the claim.
3. **Validate accuracy.** The claim is easy to look up, and you'll find numerous websites stating that it is untrue. Of course, you should also check the validity of these websites before believing them, but you'll find some are reliable sources such as NASA or well-respected news sites. We therefore conclude that the correct answer to the chapter-opening question is D—the claim is false. But let's continue with the last two steps anyway.
4. **Watch for hidden agendas.** In this case, there's no obvious hidden agenda. It seems more likely that the claim is just a misstatement of fact. Additional research will

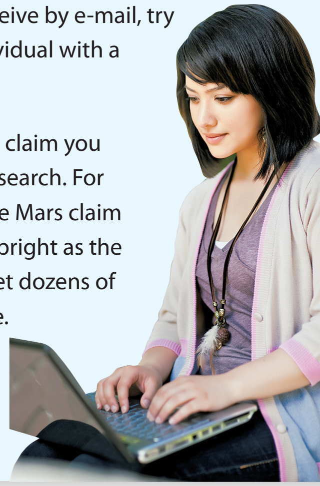
IN YOUR WORLD

Web Searches to Verify Web Sources

While some information on the Web is inaccurate or biased, the Web is also a great source for checking the accuracy of information. A good way to start is with "fact checking" websites, as long as you also verify that the fact checkers have a reputation for fairness and accuracy. A few reputable fact-checking sites include:

- For political fact checking: FactCheck.org, supported by the nonpartisan and nonprofit Annenberg Public Policy Center; PolitiFact.com, from the Tampa Bay Times; and "The Fact Checker," a blog hosted on the Washington Post website.
- For rumors, urban myths, and other odd claims, Snopes.com has a solid reputation for accuracy.
- To check the validity of messages you receive by e-mail, try TruthOrFiction.com, run by a private individual with a reputation for fairness and accuracy.

If none of those sources has covered the claim you are investigating, try a plain language Web search. For example, if you type the first sentence of the Mars claim ("On August 27, Mars will look as large and bright as the full Moon . . .") into a search engine, you'll get dozens of hits that discuss the claim and why it is false. Of course, if your search turns up conflicting claims about accuracy, you'll still need to decide which claims to believe.



show that the claim originally arose in 2003, when on August 27 Mars came slightly closer to Earth than it will come again for at least 200 years. However, Mars was still nowhere near as large and bright in our sky as the full Moon.

5. **Don't miss the big picture.** This step asks us to stand back and think about whether the claim make sense, which you can do by thinking about the hint in the chapter opener: Mars is a planet orbiting the Sun, while the Moon orbits Earth. This fact means that the Moon is always much closer to us than Mars; in fact, even at its closest, Mars is about 150 times as far from Earth as the Moon. You can then conclude that Mars could never appear as large and bright in our sky as the full Moon. (If you want to be more quantitative: At 150 times the distance of the Moon, Mars would have to be 150 times as large as the Moon in diameter in order to appear equally large in our sky. However, Mars is only about twice as large in diameter as the Moon.)

▶ **Now try Exercises 21–24.**

Quick Quiz 1A

Choose the best answer to each of the following questions. Explain your reasoning with one or more complete sentences.

1. A logical argument always includes
   a. at least one premise and one conclusion.
   b. at least one premise and one fallacy.
   c. at least one fallacy and one conclusion.
2. A fallacy is
   a. a statement that is untrue.
   b. a heated argument.
   c. a deceptive argument.
3. Which of the following could *not* qualify as a logical argument?
   a. a series of statements in which the conclusion comes before the premises
   b. a list of premises that do not lead to a conclusion
   c. a series of statements that generate heated debate
4. An argument in which the conclusion essentially restates the premise is an example of
   a. circular reasoning.
   b. limited choice.
   c. logic.
5. The fallacy of appeal to ignorance occurs when
   a. the fact that a statement p is true is taken to imply that the opposite of p must be false.
   b. the fact that we cannot prove a statement p to be true is taken to imply that p is false.
   c. a conclusion p is disregarded because the person who stated it is ignorant.

6. Consider the argument "I don't support the President's tax plan because I don't trust his motives." What is the conclusion of this argument?
   a. I don't trust his motives.
   b. I don't support the President's tax plan.
   c. The President is not trustworthy.
7. Consider again the argument "I don't support the President's tax plan because I don't trust his motives." This argument is an example of
   a. a well-reasoned, logical argument.
   b. an argument that uses the fallacy of personal attack.
   c. an argument that uses the fallacy of appeal to emotion.
8. Consider the argument "Your lack of enthusiasm for soccer proves that you are not a sports fan." This argument is an example of
   a. a well-reasoned, logical argument.
   b. an argument that uses the fallacy of diversion.
   c. an argument that uses the fallacy of limited choice.
9. Suppose that the fact that an event *A* occurs before event *B* is used to conclude that *A* caused *B*. This is an example of
   a. a well-reasoned, logical argument.
   b. an argument that uses the fallacy of false cause.
   c. hasty generalization.
10. When we speak of a *straw man* in an argument, we mean
   a. a misrepresentation of someone else's idea or belief.
   b. a person who has not used good logic.
   c. an argument so weak that it is as if it were made of straw.

Exercises 1A

REVIEW QUESTIONS

1. What is logic? Briefly explain how logic can be useful.
2. How do we define *argument*? What is the basic structure of an argument?
3. What is a fallacy? Choose three examples of fallacies from this unit, and, in your own words, describe how the given argument is deceptive.
4. Summarize the five steps given in this unit for evaluating media information, and explain how you can apply them.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

5. Mike and Erica couldn't have had an argument, because they weren't shouting at each other.
6. I will become the richest person in the world because I work in the software industry in America.
7. I didn't believe the premises on which he based his argument, so he clearly didn't convince me of his conclusion.
8. She convinced me she was right, even though she stated her conclusion before supporting it with any premises.
9. I disagree with your conclusion, so your argument must contain a fallacy.
10. Even though your argument contains a fallacy, your conclusion is believable.

BASIC SKILLS & CONCEPTS

11–20: Analyzing Fallacies. Consider the following examples of fallacies.

a. Identity the premise(s) and conclusion of the argument.

b. Briefly describe how the stated fallacy occurs in the argument.

c. Make up another argument that exhibits the same fallacy.

11. (Appeal to popularity) Apple's iPhone outsells all other smart phones, so it must be the best smart phone on the market.
12. (False cause) I became sick just hours after eating at Burger Hut, so its food must have made me sick.
13. (Appeal to ignorance) Decades of searching have not revealed life on other planets, so life in the universe must be confined to Earth.
14. (Hasty generalization) I saw three people use food stamps to buy expensive steaks, so abuse of food stamps must be widespread.
15. (Limited choice) He reads a lot of crime fiction novels, so he must be a criminal investigator.
16. (Appeal to emotion) Each year, thousands of innocent people are killed by terrorists, so it's time to take firm steps to curb terrorism.
17. (Personal attack) Senator Smith's bill on agricultural policy is a sham, because he is supported by companies that sell genetically modified crop seeds.
18. (Circular reasoning) Illegal immigration is against the law, so illegal immigrants are criminals.

19. (Diversion) Good grades are needed to get into college, and a college diploma is necessary for a good career. Therefore, attendance should count in high school grades.

20. (Straw man) The mayor wants to raise taxes to fund social programs, so she must not believe in the value of hard work.

21–24: Media Claims. Each of the following claims can easily be checked on the Web. Do a check, state whether the claim is true or false, and briefly explain why.

21. Bank of America won't let people purchase firearms with their debit or credit cards.

22. Some high-end perfumes contain ambergris, which comes from sperm whale feces and vomit.

23. Actor Nicholas Cage died in a snowboarding accident on January 17, 2013.

24. A woman named Irena Sendler was nominated for the Nobel Peace Prize for saving 2,500 Polish Jews from the Holocaust.

FURTHER APPLICATIONS

25–40: Recognizing Fallacies. In the following arguments, identify the premise(s) and conclusion, explain why the argument is deceptive, and, if possible, identify the type of fallacy it represents.

25. Obesity among Americans has increased steadily, as has the sale of video games. It follows that video games are compromising the health of Americans.

26. The polls show the Republican candidate leading by a 2-to-1 margin, so you should vote for the Republican.

27. All the mayors of my hometown have been men, which shows that men are better qualified for high office than women.

28. My father tells me that I should exercise daily. But he never exercised when he was young, so I see no need to follow his advice.

29. My baby was vaccinated and later developed autism, which is why I believe that vaccines cause autism.

30. The state has no right to take a life, so the death penalty should be abolished.

31. Everyone loves Shakespeare, because his plays have been read for many centuries.

32. Claims that GMO foods are unsafe are ridiculous, as I've never heard of anyone getting sick from them.

33. I will not give money to the earthquake relief effort. After I last gave to a charity, an audit showed that most of the money was used to pay its administrators in the front office.

34. It's not surprising that President Obama's budget contains spending increases. Democrats don't care about taxpayers' money.

35. The Congressperson is a member of the National Rifle Association, so I'm sure she will not support a ban on assault rifles.

36. My three friends who drink wine have never had heart attacks. My two friends who have had heart attacks are non-drinkers. Drinking wine is clearly a good therapy.

37. Responding to Republicans who want to end the estate tax, which falls most heavily on the wealthy, a Democrat says, "The Republicans think that rich people aren't rich enough."

38. The Wyoming toad has not been seen outside of captivity since 2002, so it must be extinct in the wild.

39. My mother reads the newspaper every day and my father visits church regularly, so it is not true that women are traditional and men are interested in politics.

40. Responding to Democrats who want to raise the fuel efficiency standards for new cars and trucks, a Republican says, "Democrats think that government is the solution to all our problems . . . .

41–44: Additional Fallacies. Consider the following fallacies (which are not discussed in the text). Explain why the fallacy applies to the example and create your own argument that displays the same fallacy.

41. The *fallacy of division* has this form:

Premise:	*X* has some property.
Conclusion:	All things or people that belong to *X* must have the same property.
Example:	Americans use more gasoline than Europeans, so Jake, who is an American, must use more gasoline than Europeans.

42. The *gambler's fallacy* has this form:

Premise:	*X* has been happening more than it should.
Conclusion:	*X* will come to an end soon.
Example:	It has rained for 10 days, which is unusual around here. Tomorrow will be sunny.

43. The *slippery slope fallacy* has this form:

Premise:	*X* has occurred and is related to *Y*.
Conclusion:	*Y* will inevitably occur.
Example:	America has sent troops to three countries recently. Before you know it, we will have troops everywhere.

44. The *middle ground fallacy* has this form:

Premise:	*X* and *Y* are two extreme positions on a question.
Conclusion:	*Z*, which lies between *X* and *Y*, must be correct.
Example:	Senator Peters supports a large tax cut, and Senator Willis supports no tax cut. That means a small tax cut must be best.

IN YOUR WORLD

45. **Evaluating Media Information.** Choose a current topic of policy discussion (examples might include gun control, health care, tax policy, or many other topics). Find a website that argues on one side or other of the topic. Evaluate the arguments based on the five steps for evaluating media information given in this unit. Write a short report on the site you visited and your conclusions about the reliability of its information.

46. **Snopes.** Visit the Snopes.com website and choose one topic from its list of the "hottest urban legends." In one or two paragraphs, summarize the legend, whether it is true or false, and why.

47. Locate data, from the web if needed, for the most recent pre-election poll conducted in your country. Analyze the results predicted by the poll in comparison with the actual results. Comment on the accuracy of the poll.

48. **Fallacy Websites.** There are many websites devoted to the study of fallacies. Visit one, and choose a fallacy of a type not covered in this unit. Explain the fallacy, and give an example of it.

49. **Editorial Fallacies.** Examine editorials and letters to the editor in your local newspaper. Find at least three examples of fallacies. In each case, describe how the argument is deceptive. If the fallacy represents one or more of the common types described in this unit, state the type.

50. **Fallacies in Advertising.** Pick a single night and a single commercial television channel, and analyze the advertisements shown over a one-hour period. Describe how each advertisement tries to persuade the viewer, and discuss whether the argument is fallacious. What fraction of the advertisements involve fallacies? Are any fallacy types more common than others?

51. **Fallacies in Politics.** Discuss the tactics used by both sides in a current or recent political campaign. How much of the campaign is/was based on fallacies? Describe some of the fallacies. Overall, do you believe that fallacies influenced (or will influence) the outcome of the vote?

52. **Personal Fallacies.** Describe an instance in which you were persuaded of something that you later decided was untrue. Explain how you were persuaded and why you later changed your mind. Did you fall victim to any fallacies? If so, how might you prevent the same thing from happening in the future?

UNIT 1B Propositions and Truth Values

"Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

—*Lewis Carroll, Through the Looking Glass*

Having discussed fallacies in Unit 1A, we now study proper arguments. The building blocks of arguments are called **propositions**—statements that make (propose) a claim that may be either true or false. A proposition must have the structure of a complete sentence and must make a distinct assertion or denial. For example:

- *Joan is sitting in the chair* is a proposition because it is a complete sentence that makes an assertion.
- *I did not take the pen* is a proposition because it is a complete sentence that makes a denial.
- *Are you going to the store?* is not a proposition because it is a question. It does not assert or deny anything.
- *three miles south of here* is not a proposition because it does not make any claim and is not a complete sentence.
- $7 + 9 = 2$ is a proposition, even though it is false. It can be read as a complete sentence, and it makes a distinct claim.

▶ Now try Exercises 13–18.

Definition

A **proposition** makes a claim (either an assertion or a denial) that may be either true or false. It must have the structure of a complete sentence.

Negation (Opposites)

The opposite of a proposition is called its **negation**. For example, the negation of *Joan is sitting in the chair* is *Joan is not sitting in the chair,* and the negation of $7 + 9 = 2$ is $7 + 9 \neq 2$. If we represent a proposition with a letter such as p, then its negation is *not* p (sometimes written $\sim p$). Negations are also propositions, because they have the structure of a complete sentence and may be either true or false.

A proposition has a **truth value** of either true (T) or false (F). If a proposition is true, its negation must be false, and vice versa. We can represent these facts with a simple **truth table**—a table that has a row for each possible set of truth values. The following truth table shows the possible truth values for a proposition p and its negation *not* p. It has two rows because there are only two possibilities.

p	*not* p	
T	F	← This row shows that if p is true (T), *not* p is false (F).
F	T	← This row shows that if p is false (F), *not* p is true (T).

Definitions

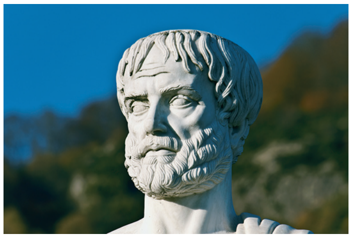
Any proposition has two possible **truth values:** T = true or F = false.

The **negation** of a proposition p is another proposition that makes the opposite claim of p. It is written *not* p (or $\sim p$) and has the opposite truth value of p.

A **truth table** is a table with a row for each possible set of truth values for the propositions being considered.

HISTORICAL NOTE

The Greek philosopher Aristotle (384–322 B.C.E.) made the first known attempt to put logic on a rigorous foundation. He believed that truth could be established from three basic laws: (1) A thing is itself. (2) A statement is either true or false. (3) No statement is both true and false. Aristotle's laws were used by Euclid (c. 325–270 B.C.E.) to establish the foundations of geometry, and logic remains an important part of mathematics.



EXAMPLE 1 Negation

Find the negation of the proposition *Amanda is the fastest runner on the team.* Write its negation. If the negation is false, is Amanda really the fastest runner on the team?

Solution The negation of the given proposition is *Amanda is not the fastest runner on the team.* If the negation is false, the original statement must be true, meaning that Amanda *is* the fastest runner on the team. ▶ **Now try Exercises 19–22.**

Double Negation

The Groucho Marx quotation in the margin may be an extreme example, but many everyday statements contain double (or multiple) negations. We've already seen that the negation *not* p has the opposite truth value of the original proposition p. The double negation *not not* p must therefore have the same truth value as the original proposition p. We can show this fact with a truth table. The first column contains the two possible truth values for p. Two additional columns show the corresponding truth values for *not* p and *not not* p.

p	*not* p	*not not* p
T	F	T
F	T	F

I cannot say that I do not disagree with you.
—*Groucho Marx*

In ordinary language, double negations rarely involve phrases like "not not," so we must analyze wording carefully to recognize them.

EXAMPLE 2 Radiation and Health

After reviewing data showing an association between low-level radiation and cancer among older workers at the Oak Ridge National Laboratory, a health scientist from the University of North Carolina (Chapel Hill) was asked about the possibility of a similar association among younger workers at another national laboratory. He was quoted as saying (*Boulder Daily Camera*):

My opinion is that it's unlikely that there is no association.

Does the scientist think there is an association between low-level radiation and cancer among younger workers?

Solution Because of the words "unlikely" and "no association," the scientist's statement contains a double negation. To see the effects of these words clearly, let's start with a simpler proposition:

p = *it's likely that there is an association (between low-level radiation and cancer)*

The word "unlikely" gives us the statement *it's unlikely that there is an association*, which we identify as *not p*. The words "no association" transform this last statement into the original statement, *it's unlikely that there is no association*, which we recognize as *not not p*. Because the double negation has the same truth value as the original proposition, we conclude that the scientist believes it likely that there *is* an association between low-level radiation and cancer among younger workers.

▶ Now try Exercises 23–24.

BY THE WAY

Ernesto Miranda confessed to and was convicted of a 1963 rape and kidnapping. His lawyer argued that the confession should not have been admitted as evidence because Miranda had not been told of his right to remain silent. The Supreme Court agreed and overturned his conviction. He was then retried and again convicted (on the basis of evidence besides the confession). Miranda was stabbed to death during a bar fight after his release from prison. A suspect in his killing chose to remain silent upon arrest, and police never filed charges.

EXAMPLE 3 The 2000 Miranda Ruling

In a June 2000 decision (*Dickerson v. United States*), the U.S. Supreme Court voted 7–2 to uphold the basic requirements of the 1966 Miranda decision (*Miranda v. State of Arizona*). That decision required that suspects taken into custody be informed of their constitutional rights, such as the right to remain silent and the right to legal counsel. In his majority opinion, Chief Justice William Rehnquist wrote:

. . . [legal] principles weigh heavily against overruling [Miranda] now.

According to this statement, did the Chief Justice feel that legal principles support or oppose the original Miranda decision?

Solution Analyzed carefully, the Chief Justice's statement contains a double negation. The first negation comes from the term "overruling," which alone would imply opposition to the original decision. But the statement argued *against* overruling and therefore implies that legal principles support the original Miranda decision.

▶ Now try Exercises 25–28.

Logical Connectors

Propositions are often joined together with *logical connectors*—words such as *and, or*, and *if . . . then*. For example, consider the following two propositions:

p = *The test was hard.*

q = *I got an A.*

If we join the two propositions with *and*, we get a new proposition that reads *the test was hard and I got an A*. If we join them with *or*, we get the statement *the test was hard or I got an A*. Although such statements are familiar in everyday speech, we must analyze them carefully.

And Statements (Conjunctions)

An *and* statement is called a **conjunction**. According to the rules of logic, the conjunction *p and q* is true only if p and q are *both* true. For example, the statement *the test was hard and I got an A* is true only if it *was* a hard test and you *did* get an A.

The Logic of *And*

Given two propositions p and q, the statement *p and q* is called their **conjunction**. It is true only if p and q are *both* true.

To make a truth table for the conjunction *p and q*, we analyze all possible combinations of the truth values of the individual propositions *p* and *q*. Because *p* and *q* each have two possible truth values (true or false), there are $2 \times 2 = 4$ cases to consider. The four cases become the four rows of the truth table.

Truth Table for Conjunction *p and q*

p	*q*	*p and q*	
T	T	T	← case 1: *p*, *q* both true
T	F	F	← case 2: *p* true, *q* false
F	T	F	← case 3: *p* false, *q* true
F	F	F	← case 4: *p*, *q* both false

▶ **Now try Exercises 29–30.**

Note that the *and* statement is true only in the first case shown in the table, where both individual propositions are true.

EXAMPLE 4 *And* Statements

Evaluate the truth value of the following two statements.

a. The capital of France is Paris and Antarctica is cold.
b. The capital of France is Paris and the capital of America is Madrid.

Solution

a. The statement contains two distinct propositions: *The capital of France is Paris* and *Antarctica is cold.* Because both propositions are true, their conjunction is also true.
b. The statement contains two distinct propositions: *The capital of France is Paris* and *the capital of America is Madrid.* Although the first proposition is true, the second is false. Therefore, their conjunction is false. ▶ **Now try Exercises 31–36.**

EXAMPLE 5 Triple Conjunction

Suppose you are given three individual propositions *p*, *q* and *r*. Make a truth table for the conjunction *p and q and r*. Under what circumstances is the conjunction true?

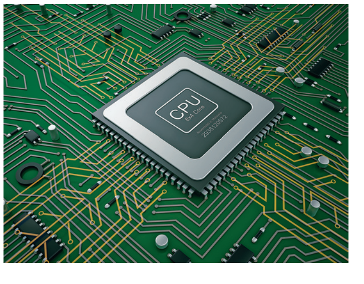
Solution We already know that the statement *p and q* has four possible cases for truth values. For each of these four cases, proposition *r* can be either true or false. Therefore, the statement *p and q and r* has $4 \times 2 = 8$ possible cases for truth values. The following truth table contains a row for each of the eight cases. Note that the four cases for *p* and *q* each appear twice, once with *r* true and once with *r* false.

p	*q*	*r*	*p and q and r*
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

The conjunction *p and q and r* is true only if all three statements are true, as shown in the first row. ▶ **Now try Exercises 37–38.**

BY THE WAY

Logical rules lie at the heart of modern computer science. Computers generally represent the numbers 0 and 1 with electric current: No current in the circuit means 0 and current in the circuit means 1. Computer scientists then think of 1 as true and 0 as false and use logical connectors to design circuits. For example, an *and* circuit allows current to pass (its value is 1 = true) only if both incoming circuits carry current.



Time Out to Think Given four propositions *p*, *q*, *r*, and *s*, how many rows are required for a truth table of the conjunction *p and q and r and s*? When is the conjunction true?

Understanding *Or*

The connector *or* can have two different meanings. If a health insurance policy says that it covers hospitalization in cases of illness *or* injury, it probably means that it covers either illness or injury *or both*. This is an example of the **inclusive *or*** that means "either or both." In contrast, when a restaurant offers a choice of soup *or* salad, you probably are not supposed to choose both. This is an example of the **exclusive *or*** that means "one or the other."

Two Types of *Or*

The word *or* can be interpreted in two distinct ways:

- An **inclusive *or*** means "either or both."
- An **exclusive *or*** means "one or the other, but not both."

In everyday life, we determine whether an *or* statement is inclusive or exclusive by its context. But in logic, we assume that *or* is inclusive unless told otherwise.

EXAMPLE 6 Inclusive or Exclusive?

Kevin's insurance policy states that his house is insured for earthquakes, fire, *or* robbery. Imagine that a major earthquake levels much of his house, the rest burns in a fire, and his remaining valuables are looted in the aftermath. Would Kevin prefer that the *or* in his insurance policy be inclusive or exclusive? Why?

Solution He would prefer an inclusive *or* so that his losses from all three events (earthquake, fire, looting) would be covered. If the *or* were exclusive, the insurance would cover only one of the losses.

▶ Now try Exercises 39–44.

Or Statements (Disjunctions)

A compound statement made with *or* is called a **disjunction.** We assume that the *or* is inclusive, so the disjunction *p or q* is true if either or both propositions are true. A disjunction *p or q* is false only if both individual propositions are false.

The Logic of *Or*

Given two propositions *p* and *q*, the statement *p or q* is called their **disjunction**. In logic, we assume that *or* is *inclusive*, so the disjunction is true if either or both propositions are true, and false only if *both* propositions are false.

These rules lead to the following truth table.

Truth Table for Disjunction *p or q*

p	*q*	*p or q*	
T	T	T	← case 1: *p*, *q* both true, so "or" is true
T	F	T	← case 2: *p* true, so "or" is true
F	T	T	← case 3: *q* true, so "or" is true
F	F	F	← case 4: *p*, *q* both false, so "or" is false

▶ Now try Exercises 45–50.

EXAMPLE 7 Smart Cows?

Consider the statement *airplanes can fly or cows can read.* Is it true?

Solution The statement is a disjunction of two propositions: (1) *airplanes can fly;* (2) *cows can read.* The first proposition is clearly true, while the second is clearly false, which makes the disjunction *p or q* true. That is, the statement *airplanes can fly or cows can read* is true.

▶ Now try Exercises 51–56.

BY THE WAY

Most search engines automatically connect words in the search box with the logical connector AND. For example, a search on *television entertainment* is really a search on *television* AND *entertainment*, and it will return any Web page that has both words, regardless of whether the words come together. If you want the exact phrase "*television entertainment,*" you can put the phrase in quotes.

If . . . Then Statements (Conditionals)

Another common way to connect propositions is with the words "if…then," as in the statement "If all politicians are liars, then Representative Smith is a liar." Statements of this type are called **conditional propositions** (or *implications*) because they propose something to be true (the *then* part of the statement) on the *condition* that something else is true (the *if* part of the statement).

We can represent a conditional proposition as *if p then q*, where proposition *p* is called the **hypothesis** (or *antecedent*) and proposition *q* is called the **conclusion** (or *consequent*).

Let's use an example to discover the truth table for a conditional proposition. Suppose that while running for Congress, candidate Jones claimed:

If I am elected, then the minimum wage will increase.

This proposition has the standard form *if p, then q*, where p = *I am elected* and q = *the minimum wage will increase.* Because each individual proposition can be either true or false, we must consider four possible cases for the truth value of *if p, then q*:

1. ***p* and *q* both true.** In this case, Jones *was* elected (*p* true) and the minimum wage increased (*q* true). Jones kept her campaign promise, so her claim, *if I am elected, then the minimum wage will increase,* was true.
2. ***p* true and *q* false.** In this case, Jones was elected, but the minimum wage did not increase. Because things did not turn out as she promised, her claim, *if I am elected, then the minimum wage will increase,* was false.
3. ***p* false and *q* true.** This is the case in which Jones was *not* elected, yet the minimum wage still increased. The conditional statement makes a claim about what should happen in the event that Jones is elected. Because she was not elected, she surely did not break any campaign promise, regardless of whether or not the minimum wage increased. It is a rule of logic that we consider Jones's claim to be true in this case.
4. ***p* and *q* both false.** Now we have the case in which Jones was not elected and the minimum wage did not increase. Again, because she was not elected, she surely did not violate her campaign promise, even though the minimum wage did not increase. As in the previous case, Jones's claim is true.

In summary, the statement *if p, then q* is true in all cases except when the hypothesis *p* is true and the conclusion *q* is false. Here is the truth table.

Truth Table for Conditional *if p, then q*

p	*q*	*if p, then q*
T	T	T
T	F	F
F	T	T
F	F	T

▶ Now try Exercises 57–58.

The Logic of *if . . . then*

A statement of the form *if p*, *then q* is called a **conditional proposition** (or *implication*). Proposition *p* is called the **hypothesis** and proposition *q* is called the **conclusion**. The conditional *if p*, *then q* is true in all cases except the case in which *p* is true and *q* is false.

Time Out to Think Suppose candidate Jones had made the following campaign promise: *If I am elected, I will personally eliminate all poverty on Earth*. According to the rules of logic for cases (3) and (4) above, we consider this statement to be true in the event that Jones is *not* elected. Is this logical definition of truth the same one that you would use if you heard her make this promise? Explain.

EXAMPLE 8 Conditional Truths

Evaluate the truth of the statement *if* $2 + 2 = 5$, *then* $3 + 3 = 4$.

Solution The statement has the form *if p*, *then q*, where *p* is $2 + 2 = 5$ and *q* is $3 + 3 = 4$. Both *p* and *q* are clearly false. However, according to the rules of logic, the conditional *if p*, *then q* is true any time *p* is false, regardless of what *q* says. Therefore, the statement *if* $2 + 2 = 5$, *then* $3 + 3 = 4$ is true. ▶ Now try Exercises 59–66.

Alternative Phrasings of Conditionals

In ordinary language, conditional statements don't always appear in the standard form *if p*, *then q*. In such cases, it can be useful to rephrase the statements in the standard form. For example, the claim *I'm not coming back if I leave* has the same meaning as *if I leave, then I'm not coming back*. Similarly, the statement *more rain will lead to a flood* can be recast as *if there is more rain, then there will be a flood.*

Two common ways of phrasing conditionals use the words *necessary* and *sufficient*. Consider the true implication *if you are living, then you are breathing*. Our understanding of this statement is that in order to be living, it is necessary to be breathing, or more briefly *breathing is necessary for living*. In general, the true implication *if p*, *then q* is equivalent to the statement *q is necessary for p*. This statement does not mean that breathing is the only necessity for living. It is one of many necessities (such as eating, breathing, and having a heartbeat).

Now consider the true implication *if you are in Denver, then you are in Colorado*. The meaning is this statement is that to be in Colorado, it is sufficient to be in Denver, or more briefly, *being in Denver is sufficient for being in Colorado*. More generally, the true implication *if p*, *then q* is equivalent to the statement *p is sufficient for q*. This statement does not mean that being in Denver is necessary for being in Colorado, because many other places are also in Colorado.

HISTORICAL NOTE

The system of logic presented in this chapter was first developed by the ancient Greeks and is called *binary logic* because a proposition is either true or false, but not both and certainly nothing in between. Today, mathematicians also use systems of logic in which other truth values are possible. One form, called *fuzzy logic,* allows for a continuous range of values between absolutely true and absolutely false. Fuzzy logic is used in many new technologies.

Alternative Phrasings of Conditionals

The following are common alternative ways of stating *if p*, *then q*:

p is sufficient for q	*p will lead to q*	*p implies q*
q is necessary for p	*q if p*	*q whenever p*

EXAMPLE 9 Rephrasing Conditional Propositions

Recast each of the following statements in the form *if p*, *then q*.

a. A rise in sea level will devastate Florida.
b. A red tag on an item is sufficient to mean it's on sale.
c. Eating vegetables is necessary for good health.

Solution

a. This statement is equivalent to *if sea level rises, then Florida will be devastated.*

b. This statement can be written as *if an item is marked with a red tag, then it is on sale.*

c. This statement can be expressed as *if a person is in good health, then the person eats vegetables.*

▶ **Now try Exercises 67–72.**

Converse, Inverse, and Contrapositive

The order of the propositions does not matter in a conjunction or disjunction. For example, *p and q* has the same meaning as *q and p*, and *p or q* has the same meaning as *q or p*. However, when we switch the order of the propositions in a conditional, we create a different proposition called the **converse**. The following box summarizes definitions for the converse and two other variations on a conditional proposition *if p, then q*.

Variations on the Conditional

Name	Form	Example
Conditional	*if p, then q*	If you are sleeping, then you are breathing.
Converse	*if q, then p*	If you are breathing, then you are sleeping.
Inverse	*if not p, then not q*	If you are not sleeping, then you are not breathing.
Contrapositive	*if not q, then not p*	If you are not breathing, then you are not sleeping.

We can determine the truth values for the converse, inverse, and contrapositive with a truth table. Because all the statements use the same two propositions (*p*, *q*), the table has four rows. The first two columns show the truth values for *p* and *q*, respectively. The next two columns show the truth values for the negations *not p* and *not q*, which are needed for the inverse and contrapositive. The fifth column shows the truth values found previously for the conditional proposition *if p, then q*. We then find the truth values for the converse, inverse, and contrapositive by applying the rule that a conditional is false only when the hypothesis is true and the conclusion is false.

logically equivalent (*if p, then q* ↔ *if not q, then not p*)

p	*p*	*not p*	*not q*	*if p, then q*	*If q, then p* (converse)	*if not p, then not q* (inverse)	*if not q, then not p* (contrapositive)
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

logically equivalent (converse ↔ inverse)

Note that the truth values for the conditional *if p, then q* are the same as the truth values for its contrapositive. We therefore say that a conditional and its contrapositive are **logically equivalent**: If one is true, so is the other, and vice versa. The table also shows that the converse and inverse are logically equivalent.

Definition

Two statements are **logically equivalent** if they share the same truth values: If one is true, so is the other, and if one is false, so is the other.