The Gen X Series

MATHS OLYMPIAD 8

Useful for Maths Olympiads Conducted at School, National & International Levels

Author
Prasoon Kumar
Peer Reviewer
Jyotsna Gopikrishnan

Strictly According to the Latest Syllabus of Maths Olympiad

V&S PUBLISHERS

Published by:



V&S PUBLISHERS

F-2/16, Ansari road, Daryaganj, New Delhi-110002
☎ 23240026, 23240027 • *Fax:* 011-23240028
Email: info@vspublishers.com • *Website:* www.vspublishers.com
Online Brandstore: ***amazon.in/vspublishers***

Regional Office : Hyderabad
5-1-707/1, Brij Bhawan (Beside Central Bank of India
Lane) Bank Street, Koti, Hyderabad - 500 095
☎ 040-24737290
E-mail: vspublishershyd@gmail.com

Branch Office : Mumbai
Jaywant Industrial Estate, 1st Floor–108, Tardeo Road
Opposite Sobo Central Mall, Mumbai – 400 034
☎ 022-23510736
E-mail: vspublishersmum@gmail.com

BUY OUR BOOKS FROM: AMAZON FLIPKART

© Copyright: V&S PUBLISHERS

ISBN 978-93-505795-9-6
New Edition

DISCLAIMER

While every attempt has been made to provide accurate and timely information in this book, neither the author nor the publisher assumes any responsibility for errors, unintended omissions or commissions detected therein. The author and publisher makes no representation or warranty with respect to the comprehensiveness or completeness of the contents provided.

All matters included have been simplified under professional guidance for general information only, without any warranty for applicability on an individual. Any mention of an organization or a website in the book, by way of citation or as a source of additional information, doesn't imply the endorsement of the content either by the author or the publisher. It is possible that websites cited may have changed or removed between the time of editing and publishing the book.

Results from using the expert opinion in this book will be totally dependent on individual circumstances and factors beyond the control of the author and the publisher.

It makes sense to elicit advice from well informed sources before implementing the ideas given in the book. The reader assumes full responsibility for the consequences arising out from reading this book.

For proper guidance, it is advisable to read the book under the watchful eyes of parents/guardian. The buyer of this book assumes all responsibility for the use of given materials and information.

The copyright of the entire content of this book rests with the author/publisher. Any infringement/transmission of the cover design, text or illustrations, in any form, by any means, by any entity will invite legal action and be responsible for consequences thereon.

Printed at : Param Offsetters, Okhla, New Delhi–110020

Publisher's Note

The current decade has firmly established V&S Publishers as one of the Leading Publishers of General Trade Mass Appeal Books across popular genres along with Academic Books for school children. Having been in publishing trade for over 40 years we understand the need of the hour when it comes to Books. After successfully publishing over 600 titles in a rather short time span of 5 years and establishing a pan India network of booksellers & distributors including ecommerce platforms viz – Amazon, Flipkart etc; an extensive market research lead us to publishing our Bestselling Series ever – OLYMPIAD BOOKS.

The Olympiad Series launched 4 years back under our GEN X SERIES Imprint gained widespread popularity amongst students and teachers immediately owing to its rich, high quality content and unique presentation. Published for Classes 1–10 across subjects English, Maths, Science & Computers, these books are holistic in nature and unlike run of the mill workbooks in the market, which are mere replicas of one another, these books deal with the content in a much comprehensive manner. Recourse to the 'Principles of Applied Psychology of Student Learning' has been utilised to upgrade levels of conceptual understanding in all designated subjects among class 1 to 10 students.

Encouraged by this huge acceptability of our Olympiad Series among parents and students and after revolutionising the way Olympiad books were written and published, we at V&S Publishers decided to take this to the next level.

We present to you Brand New Edition of our book – **MATHS OLYMPIAD CLASS 8.**

Each book originally written by Subject Matter Expert, is now further Peer Reviewed by top School Teachers and HODs to eliminate the slightest of errors that were present earlier. Furthermore to ensure authenticity and accuracy of content the book is now completely revised and reformatted as per the guidelines of the examining body. The New and Revised Olympiad Book is now suited to Olympiad examinations conducted at School Level, National Level or International Level by any and all organisations/companies.

The New Edition of this Maths Olympiad Class 8 is written in a Guide like pattern with images and illustrations at every step & is divided into different sections. Each chapter comes with Basic Theory and Solved Examples. Multiple Choice Questions with their Answer Keys and Solutions are liberally included. In order to help students become aware of and to simulate the actual exam conditions, a bunch of OMR Sheets have been enclosed with the book as well.

Amalgamation of Technology with Content has always been at the forefront for V&S Publishers and our new Student Portal for Olympiad Practice–www.vsexamprep.com is further testimony to that. We recommend students logging in and using it to their benefit.

P.S. While every care has been taken to ensure correctness of content, if you come across any error, howsoever minor, anywhere in the book, do not hesitate to discuss with your teachers while pointing that out to us in no uncertain terms.

We wish you All The Best!

Contents

SECTION 1: MATHEMATICAL REASONING

1. Playing with Numbers 9
2. Rational Numbers 13
3. Square and Square Roots 18
4. Cube and Cube Roots 22
5. Exponents and Powers 26
6. Profit and Loss 30
7. Algebraic Expressions and Their Identities 35
8. Linear Equations in One Variable 40
9. Quadrilaterals 44
10. Mensuration 49
11. Visualising Solid Shapes 53
12. Data Handling 57
13. Direct and Inverse Variations 59

SECTION 2: LOGICAL REASONING

1. Alphabet Test 64
2. Odd One Out 68
3. Coding Decoding 70
4. Direction Sense Test 74
5. Series Completion 78
6. Pattern 81
7. Number Ranking 84
8. Analytical Reasoning 87
9. Venn Diagram 91
10. Embedded Figure 95
11. Completion of Incomplete Pattern 101
12. Water Images 110
13. Figure Matrix 114

SECTION 3: ACHIEVERS' SECTION

Higher Order Thinking Skills (HOTS) 125
SHORT ANSWER QUESTIONS 129
MODEL TEST PAPER – 1 131
MODEL TEST PAPER – 2 137
HINTS AND SOLUTIONS 144

SECTION 1
MATHEMATICAL REASONING

Playing with Numbers

1

Numbers in General Form

In general, any two digit number pq made of digits p and q can be written as,

$$pq = 10 \times p + q = 10p + q$$
$$qp = 10 \times q + p = 10q + p$$

Similarly,

A 3-digit number pqr made of digits p, q and r can be written as,

$$pqr = 100\ p + 10q + r$$

Example 1: Ravi took a number pq. He reversed the digits of the number and then when he added the original number he got 187. If $p > q$, then find the values of p and q.

Solution: $pq \Rightarrow 10p + q$

After reversing the digits, $qp \Rightarrow 10q + p$

$\therefore \quad pq + qp \Rightarrow (10p + q) + (10q + p)$

$= 11p + 11q = 11(p + q)$

$\Rightarrow \quad 11(p + q) = 187$

$\Rightarrow \quad p + q = 17$

[17 can be obtained by adding 9 and 8 only].

$\because p > q, \therefore p = 9, q = 8.$

Example 2: Sangam takes a number 'ab' and reversed its digits. Afterwards, he subtracted the new number from the original number to obtain 27 as a result. Find a and b, if, $ab > ba$.

Solution: $ab \Rightarrow 10a + b$

After reversing the digits, $ba \Rightarrow 10b + a$

$\therefore \quad ab - ba \Rightarrow (10a + b) - (10b + a)$

$\Rightarrow 9(a - b) = 72$

$\Rightarrow \quad a - b = 8 \quad \ldots(i)$

$\therefore a = 9, b = 1$ only satisfies the above equation (i).

Example 3: If we take a number 'abc' and interchange the first and last digits to obtains 'cba', then,

$abc - cba = 198$

$abc + cba = 88p$

are obtained. Find p, a, b and c.

Solution: $abc = 100\ a + 10\ b + c$

$cba = 100\ c + 10\ b + a$

$\therefore \quad abc - cba = 99\ (a - c) = 198$

$\Rightarrow \quad (a - c) = 2 \quad \ldots(i)$

$abc + cba = 88p \Rightarrow 101\ (a + c) + 2ab = 800 + 80 + p$

$\Rightarrow \quad 20b = 80$

$\Rightarrow \quad b = 4,\ a + c = \dfrac{800 + p}{101} \quad \ldots(ii)$

p, a, c are natural numbers.

Therefore, $p = 8, a + c = \dfrac{808}{101} = 8 \quad \ldots(iii)$

$a = 5, c = 3, b = 4$ [From (i), (ii), (iii)].

$\therefore a = 5, b = 4, c = 3, p = 8.$

Letters For Digits

Here, some mathematical puzzles are solved using general techniques.

Example 4: Determine a, b, c, if,

$$\begin{array}{r} 4\,a \\ +\,9\,8 \\ \hline c\,b\,3 \end{array}$$

Solution: $4a = 40 + a$

$98 = 90 + 8$

$\therefore 4a + 98 = 138 + a$

$a + 8$ gives 3 on addition, if, $a = 5$

On addition, 1 is left as a carry which is added to (4 + 9).

$\therefore \quad cb = 4 + 9 + 1 = 14$

$\therefore \quad c = 1, b = 4, a = 5$

Example 5: $7a$, Find a.

$\times 6$

aaa

Solution: $7a = 70 + a$.

$\therefore \quad (70 + a) \times 6 = 420 + 6a$

$\because$ a is a natural number, and, $(420 + 6a)$ give $(a\ a\ a)$ as a result.

$\therefore\ a = 4$

Example 6: Find a, b, c, if,

$$\begin{array}{r} a\,8\,3 \\ \times\, c\,9 \\ \hline a\,0\,4\,a \\ +1\,5\,b\,b\,0 \\ \hline c\,c\,a\,0\,a \end{array}$$

Solution: $3 \times 9 = 27$

$\therefore \quad a = 7$

$\therefore$

$$\begin{array}{r} 7\,8\,3 \\ \times\, c\,9 \\ \hline 7\,0\,4\,7 \\ +1\,5\,b\,b\,0 \\ \hline c\,c\,7\,0\,7 \end{array}$$

$\because$ $4 + b$ gives such a number whose unit's place digit is zero.

$\therefore\ b = 6$. Now,

$$\begin{array}{r} 7\,0\,4\,7 \\ +1\,5\,6\,6\,0 \\ \hline 2\,2\,1\,0\,7 \end{array}$$

$\therefore\ c = 2$

Tests of Divisibility

1. **Divisibility by 10:** Numbers whose digit in one's place is zero are divisible by 10, otherwise, not.
2. **Divisibility by 2:** If the one's digit of a number is 0,2,4,6,8 then the number is divisible by 2.
3. **Divisibility by 3 and 9:**
   (i) A number N is divisible by 9, if, the sum of its digits is divisible by 9.
   (ii) A number N is divisibility by 3, if, the sum of its digits is divisible by 3.
4. **Divisibility by 5:** If the one's digit of a number is 0 or 5, then the number is divisible by 5.
5. **Divisibility by 11:** A number $abcdefg$ is divisible by 11, if,
   $(a + c + e + g) - (b + d + f) = 0$,

Multiple Choice Questions

1. What is the next number in the series 5, 10, 26, 50, 122, ……
   (a) 129 (b) 170 (b) 204 (d) 138
2. A number is divisible by 63 if it is divisible by:
   (a) 7 (b) 9
   (c) both 7 and 9 (d) both 3 and 7
3. What is the sum of first 15 natural numbers?
   (a) 122 (b) 106 (c) 105 (d) 120
4. 7,835 + 2b1 = 8126, then the value of b will be:
   (a) 9 (b) 8 (c) 7 (d) 3
5. What is the sum of first 20 odd numbers?
   (a) 350 (b) 400 (c) 355 (d) 420
6. $\begin{array}{r} 1\,8\,2 \\ \times\ 2\,2 \\ \hline a\,0\,0\,a \end{array}$, the value of '$a$' will be:
   (a) 2 (b) 3 (c) 4 (d) 8
7. What will be the last digit of 7^{333} ?
   (a) 1 (b) 7 (c) 3 (d) 9
8. Which of the following numbers is divisible by 11?
   (a) 1221 (b) 1223 (c) 1332 (d) 1343
9. $\begin{array}{r} 7\,3\,x\,5 \\ -\ 2\,y\,7\,7 \\ \hline 4\,5\,1\,8 \end{array}$, the value of $(x + y)$ will be:
   (a) 16 (b) 17 (c) 18 (d) 15
10. $\begin{array}{r} a\,a \\ \times\ a\,a \\ \hline b\,8\,b \end{array}$ Given $1 \le a, b \le 9$,
    then the value of (ab) will be:
    (a) 32 (b) 16
    (c) 8 (d) 4
11. 763*312, which number should the * be replaced with to make the number divisible by 9?
    (a) 7 (b) 5
    (c) 8 (d) 6
12. 76215*, if the replacement of * by a number gives a number which is divisible by 11, the number will be:
    (a) 8 (b) 7 (c) 6 (d) 9
13. $\begin{array}{r} A\,B\,C \\ A\,B\,C \\ +\ A\,B\,C \\ \hline B\,B\,B \end{array}$, the values of A, B, C are digits from 1 to 9. What will be value of B?
    (a) 8 (b) 4 (c) 1 (d) 3
14. What will be the sum of first 22 even natural numbers?
    (a) 506 (b) 406 (c) 484 (d) 253
15. One candle was guaranteed to burn for 6 hours, the other for 2 hours. They were both lit at same time. After some time one candle was twice as long as the other. For how long had they been burning?
    (a) 3 hours (b) $\frac{6}{5}$ hours
    (c) $\frac{4}{3}$ hours (d) $\frac{3}{2}$ hours
16. Find a 3-digit number, such that all its digits are prime and the 3 digits are the factors of the number?
    (a) 735 (b) 537 (c) 435 9 (d) 245
17. Complete the square given below and find the value of the sum of missing numbers. The sum of the magic square is 34.

5	x	e	d
16	y	7	c
a	13	b	6
2	z	9	f

    (a) 68 (b) 39 (c) 78 (d) 84

18. Three numbers are such that their sum is 10 and their product is maximum. The product will be:
(a) 32 (b) 36 (c) 45 (d) 42

19. What will be the one's place digit of 6^{222}?
(a) 4 (b) 8
(c) 1 (d) 6

20. Find the smallest number which can be expressed as the sum of two cubes of natural numbers.
(a) 1729 (b) 1001 (c) 1728 (d) 1332

21. $\begin{array}{r} PAT \\ +EAT \\ \hline FEEA \end{array}$, where, P, A, T, E, F are digits from 1 to 9 what will be the value of F?
(a) 4 (b) 3 (c) 2 (d) 1

22. Sum of 3 numbers = product of these 3 numbers. If the numbers are consecutive and natural, find the triplet having least value for their sum.
(a) 2, 3, 4 (b) 1, 2, 3
(c) 3, 4, 6 (d) 1, – 1, 0

23. The square of a number is having 5 at its units place and 2 at its tenths place, then the least natural number having these properties are:
(a) 5 (b) 15 (c) 25 (d) 4

24. The product 135×135 will be equal to:
(a) 19625 (b) 16925 (c) 18225 (d) 16235

25. Which of the following number is not a perfect squares ?
(a) 1024 (b) 441 (c) 1681 (d) 1282

26. $26 + 34 \times 17 \div 4 = 34$, which of the two signs should be interchanged to get the desired result?
(a) No change (b) ÷, +
(c) ×, ÷ (d) +, ×

27. Which is the least number divisible by 2, 3, 5 and 55 ?
(a) 110 (b) 550 (c) 660 (d) 330

28. What is the square number just greater than 60, which can be expressed as a sum of two successive triangular numbers?
(a) 72 (b) 64 (c) 81 (d) 100

29. What will be one's place digit for 9^{201}?
(a) 9 (b) 1 (c) 3 (d) 7

30. What is the value of P if P, Q, R are replaced by any digits from 1 to 9 such that $PQ \times QP = RQPR$.
(a) 6 (b) 8 (c) 7 (d) 5



Rational Numbers

2

Rational Numbers

A number in the form of $\frac{a}{b}$, where, 'a' and 'b' are integers, and $b \neq 0$, is called a rational number.

Example: $\frac{-1}{2}, \frac{3}{5}, \frac{4}{90}$, etc.

Equivalent Rational Numbers

Two rational numbers $\frac{m}{n}$ and $\frac{p}{q}$ are equivalent, if $m \times q = n \times p$.

Infinite number of rational numbers, each of which is equivalent to a given rational number, can be formed. The equivalent rational numbers can be obtained by multiplying the numerator and denominator of the given rational number by the same non-zero integer.

$\therefore \quad \frac{a}{b} = \frac{a \times p}{b \times p} = \frac{a \times q}{b \times q}$ (where, $p \neq 0$ and q $\neq 0$).

A rational number, whose numerator and denominator has only one common factor equal to 1 is said to be standard form of a rational number.

Terminating and Non-Terminating Decimals

If the denominator of a rational number has no factors other than 2 or 5 or both it is called terminating decimal, otherwise, the decimal will be non-terminating decimal.

Example: $\frac{1}{5}, \frac{1}{25}, \frac{1}{50}, \frac{1}{100}$ are terminating decimals and $\frac{1}{70}, \frac{70}{85}, \frac{14}{75}, \frac{17}{55}$ are non-terminating decimals.

While checking for terminating or non-terminating decimal first reduce the rational number into its standard form.

Example 1: Which of the following rational numbers is the smallest ?

(a) $\frac{-15}{7}$, (b) $\frac{-5}{28}$,

(c) $\frac{-25}{49}$, (d) $\frac{-35}{42}$.

Solution: LCM of denominators, *i.e.,* 7,28,49,42 is 588.

Now
$$\frac{-15}{7} = \frac{-15 \times 84}{7 \times 84} = \frac{-1260}{588}$$
$$\frac{-5}{28} = \frac{-5 \times 21}{28 \times 21} = \frac{-105}{588}$$
$$\frac{-25}{49} = \frac{-25 \times 12}{49 \times 12} = \frac{-300}{588}$$
$$\frac{-35}{42} = \frac{-35 \times 14}{42 \times 14} = \frac{-490}{588}$$

$\because \quad \frac{-1260}{588}$ is the smallest of the four rational numbers.

$\therefore \quad \frac{-15}{7}$ is the smallest rational number among the given rational numbers.

Example 2: The sum of two rational numbers is $-$. If one of them is $\frac{1}{4}$, find the other number.

Solution: Let the other rational number be x.

$\therefore \quad \frac{1}{4} + x = \frac{7}{8}$

$\Rightarrow \quad x = \frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

$\therefore$ Required rational number = 5/8.

Example 3: With what rational number should $\frac{-25}{343}$ be multiplied to get $\frac{5}{14}$ as quotient (product)?

Solution: Let required number be x,

$$\therefore \quad \frac{-25}{343} \times x = \frac{5}{14}$$

$$\Rightarrow \quad x = \frac{5}{14} \times \frac{-343}{25} = \frac{-49}{10}$$

$\therefore$ Required number $= \frac{-49}{10}$

Example 4: Simplify:

$$\frac{7}{12} \times \frac{28}{3} - \frac{7}{10} \times \frac{5}{13} + \frac{2}{13} \times 169$$

Solution: We have

$$\frac{7}{12} \times \frac{28}{3} - \frac{7}{10} \times \frac{5}{13} + \frac{2}{13} \times 169$$

$$= \frac{7}{12} \times \frac{28}{13} - \frac{7 \times 5}{10 \times 13} + 26$$

$$= \frac{7 \times 14}{6 \times 13} - \frac{7}{26} + 26 = \frac{7}{26}\left(\frac{14}{3} - 1\right) + 26$$

$$= \frac{7}{26} \times \frac{11}{3} + 26 = \frac{77}{78} + 26 = \frac{77 + 2028}{78} = \frac{2105}{78}$$

Properties of Rational Numbers

1. **Commutative Property of Addition:** If x and y are two rational numbers, then $x + y = y + x$.
2. **Associative Property of Addition:** Let x,y and z be any three rational numbers, then
$$(x + y) + z = x + (y + z).$$
3. **Property of Zero:** Let x be any rational number, then,
$$0 + x = x + 0 = x, \text{ and,}$$
$$0 \times x = x \times 0 = 0.$$
4. **Additive/Multiplicative Inverse Property:** Let x be any rational number then, $(-x)$ is called the additive inverse of x, as.
$$x + (-x) = (-x) + x = 0.$$
Let x $(x \neq 0)$ be any rational number, then, $\left(\frac{1}{x}\right)$ is called the multiplicative inverse of x, as.
$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1.$$
5. **Associative Property of Multiplication:** Let x, y and z be three rational numbers then,
$$x \times (y \times z) = (x \times y) \times z.$$
7. **Distributive Property:** Let x, y and z be any three rational numbers, then,

$x \times (y + z) = xy + xz$, or, $x \times (y - z) = xy - xz$.

8. **Property of 1:** Let x be any rational number, then $x \times 1 = 1 \times x = x$.

Example 5: Let x, y, z be any three rational numbers, then which of the following alternatives is not true?

(a) $(x + y) + z = (x + y) + (y + z) - y$

(b) $x \div (y \div z) = (x \div y) \div (y \div z)$

(c) $(x + y) \times z =$ (x - y) z + 2 (y × z)

(d) $(x + y) \div z = (x \div z) + (y \div z)$

Solution:

$\therefore$ Option (b) is not true.

Example 6: $x \times 0 = 0 \times x$, is true, but $x \div 0 = 0 \div x$, is true or false? Justify.

Solution: $x \times 0 = 0 \times x = 0$ is satisfied by property of zero, but, if,

$x \div 0 = 0 \div x$, then, in L.H.S., 0 comes in denominator, which violates the condition of rational number. So, that statement is false.

Rational Numbers between Two Rational Numbers

- Between any two rational numbers, there exists infinitely many rational numbers.
- Let 'a' and 'b' be two rational numbers, such that, $a < b$, then,

$$a < \frac{a + b}{2} < b.$$

▪ Unlike natural numbers and integers, rational numbers do not have successors and predecessors.

Example 7: Find two rational numbers between $\frac{2}{13}$ and $\frac{5}{3}$.

Solution: LCM of 3 and 13 = $3 \times 13 = 39$.

$$\therefore \quad \frac{2}{13} = \frac{2 \times 3}{13 \times 3} = \frac{6}{39}$$

$$\frac{5}{13} = \frac{5 \times 13}{3 \times 13} = \frac{65}{39}$$

Between the two integers 6 and 65, there are, $65 - 6 - 1 = 58$ integers,

$\therefore$ We choose any two integers, say, 8, and 23, then,

$$\frac{6}{39} < \frac{8}{39} < \frac{23}{39} < \frac{65}{39}$$

$\therefore$ Two integers between $\frac{2}{13}$ and $\frac{5}{3}$ are $\frac{8}{39}$ and $\frac{23}{39}$.

Example 8: Find ten rational numbers between $\frac{2}{3}$ and 3.

Solution: LCM of 3 and $1 = 3 \times 1 = 3$.

$$\frac{2}{3} = \frac{2}{3}, \text{ and } \frac{3}{1} = \frac{3 \times 3}{1 \times 3} = \frac{9}{3}$$

Between the two integers 2 and 9, there are, $9 - 2 - 1 = 6$ integers, but, we require 10 rational numbers.

$\therefore$ we multiply the common denominator by any natural number, which is greater than 1, say 2, then,

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \text{ and } \frac{3}{1} = \frac{9 \times 2}{3 \times 2} = \frac{18}{6}$$

$\therefore$ Ten rational numbers between $\frac{2}{3}$ and 3 are

$$\frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}, \frac{12}{6}, \frac{13}{6}, \frac{14}{6} \text{ or}$$

$$\frac{5}{6}, 1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2, \frac{13}{6} \text{ and } \frac{7}{3}$$

Multiple Choice Questions

1. If $\frac{3}{5}$ of a number exceeds its $-$ by 44, then what is the number?
   (a) 144 (b) 148 (c) 140 (d)160

2. A bus is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance will it cover in $7\frac{1}{2}$ hours?
   (a) 423 km (b) 453 km
   (c) 443 km (d) 463 km.

3. The sum of two numbers is $\frac{-4}{3}$. If one of them is –5 then what is the other number?
   (a) $\frac{11}{3}$ (b) $\frac{-11}{3}$ (c) $\frac{16}{3}$ (d) $\frac{19}{3}$

4. In a school $\frac{5}{8}$ of the students are boys. If the number of girls are 270, what is the number of boys in the school?
   (a) 440 (b) 450 (c) 420 (d) 400

5. A cord of length $58\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?
   (a) $2\frac{1}{4}$ m (b) $\frac{32}{75}$ m (c) $\frac{64}{75}$ m (d) $\frac{8}{15}$ m

6. The product of two numbers is $\frac{-16}{35}$. If one of the number is $\frac{-15}{14}$ what is the other number?
   (a) $\frac{16}{75}$ (b) $\frac{32}{75}$ (c) $\frac{64}{75}$ (d) $\frac{8}{15}$

7. What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?
   (a) $\frac{-5}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) $\frac{-5}{4}$

8. What is additive inverse of $\frac{-7}{9}$?
   (a) $\frac{7}{9}$ (b) $\frac{-9}{7}$ (c) $\frac{9}{7}$ (d) 1

9. The sum of two rational numbers is – 3. If one of the number is $\frac{-10}{3}$ what is the other number?
   (a) $\frac{1}{3}$ (b) $\frac{13}{3}$ (c) $\frac{19}{3}$ (d) $\frac{-19}{3}$

10. The cost of $7\frac{1}{2}$ metres of cloth is ₹ $78\frac{3}{4}$. What is the cost of one metre of cloth?
    (a) ₹ $13\frac{1}{2}$ (b) ₹ $10\frac{1}{2}$ (c) ₹ $16\frac{1}{2}$ (d) ₹ $12\frac{1}{2}$

11. By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?
    (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

12. By what rational number should we multiply $\frac{-16}{63}$ to get $\frac{-4}{7}$.
    (a) $\frac{7}{4}$ (b) $\frac{9}{4}$ (c) $\frac{3}{4}$ (d) $\frac{13}{4}$

13. What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?
    (a) $\frac{75}{72}$ (b) $\frac{85}{72}$ (c) $\frac{83}{72}$ (d) $\frac{95}{72}$

14. What is reciprocal of $\left(\frac{1}{2}+\frac{1}{5}\right)$?
    (a) $\frac{7}{10}$ (b) $\frac{10}{7}$ (c) $\frac{-7}{10}$ (d) $\frac{-10}{7}$

15. Which rational number is in between $\frac{-2}{3}$ and $\frac{-1}{4}$?
    (a) $\frac{-5}{24}$ (b) $\frac{}{12}$
    (c) $\frac{5}{12}$ (d) None of these

16. What is the reciprocal of $\left(\frac{1}{5}\times\frac{2}{5}\div\frac{4}{5}\right)$?

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) 10 (d) 5

17. What should be added to $\frac{-3}{5}$ to get $\frac{-1}{3}$?

(a) $\frac{4}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{15}$ (d) $\frac{8}{15}$

18. What is the additive inverse of $\left(\frac{3}{4}-\frac{2}{3}+\frac{1}{5}\right)$?

(a) $\frac{17}{60}$ (b) $\frac{-17}{60}$ (c) $\frac{60}{17}$ (d) $\frac{-60}{17}$

19. What is the value of $\frac{3}{4}\div\frac{5}{8}\times\frac{3}{7}+\frac{2}{9}-\frac{1}{3}$?

(a) $\frac{127}{315}$ (b) $\frac{117}{315}$

(c) $\frac{107}{315}$ (d) None of these

20. What is the value of

$$2-\left[5-\left\{4-\frac{3}{2}\left(2-\frac{2}{3}\right)\right\}\right]?$$

(a) 1 (b) –1 (c) 2 (d) –2

21. How many rational numbers lie between $\frac{1}{5}$ and $\frac{1}{3}$?

(a) One (b) Two
(c) Three (d) Infinite

22. If x is a non-zero rational number, then what is the value of x°?

(a) 1 (b) 0
(c) –1 (d) Not defined .

23. Which number is largest among

$$\frac{4}{-9},\frac{-5}{12},\frac{7}{-18},\frac{-2}{3}$$

(a) $\frac{7}{-18}$ (b) $\frac{4}{-9}$ (c) $\frac{-5}{12}$ (d) $\frac{-2}{3}$

24. Which number is smallest among

$$\frac{-7}{12},\frac{-5}{6},\frac{13}{-18},\frac{23}{-24}$$

(a) $\frac{-5}{6}$ (b) $\frac{-7}{12}$ (c) $\frac{-5}{6}$ (d) $\frac{23}{-24}$

25. Closure property of rational number will be valid for _____.

(a) Addition (b) Subtraction
(c) Multiplication (d) All of these.

26. The difference between two numbers is 22 and their product is 240, what is the difference between their reciprocals ?

(a) $\frac{11}{120}$ (b) $\frac{-11}{120}$ (c) $\frac{11}{109}$ (d) $\frac{-11}{109}$

27. If $a : b = 2 : 3$, then what will be the value of $\frac{a+b}{a-b}$?

(a) $\frac{-3}{2}$ (b) $\frac{3}{2}$ (c) –5 (d) $\frac{-1}{5}$

28. If a : b = 1:2, then $\frac{(a+b)^2}{ab}=$

(a) $\frac{9}{2}$ (b) $\frac{9}{8}$ (c) $\frac{9}{4}$ (d) $\frac{1}{4}$

29. The sum of two numbers 'p' and 'q' is 16 and their product is 48, then, $\left(\frac{1}{p^2}\right)^2+\left(\frac{1}{q^2}\right)^2=$

(a) 2.06×10^{-4} (b) $\frac{3}{16}$

(c) 4.12×10^{-5} (d) 3.9545×10^{-3}

30. The sum of two numbers is 24 and their difference is 10, then, value of $\frac{x}{y}=$ [where x > y].

(a) $\frac{27}{18}$ (b) $\frac{17}{7}$ (c) $\frac{118}{16}$ (d) $\frac{14}{34}$



Squares and Square Roots

3

Square

Square means product of the number with the number itself.

Example: $x^2 = x \times x$

$5^2 = 5 \times 5 = 25$

Perfect Square

A natural number is called a perfect square, if it is the square of some natural number.

Example 1: Find the number whose square is 1764.

Solution:

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$= 2^2 \times 3^2 \times 7^2$

$= (42)^2$

Example 2: By what least number should 6300 be divided to get a perfect square number.

Solution: $6300 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

To make 6300 a perfect square, it should be divided by 7. As 7 is alone in above prime factors.

Some Properties of Perfect Squares

I. A number ending in 2, 3, 7, 8, is never a perfect square.

Example: 22, 113, 257, 218 are not perfect squares.

II. The squares of an even number is always even.

Example: $8^2 = 64$, $12^2 = 144$, $18^2 = 324$

III. The squares of an odd number is always odd.

Example: $7^2 = 49$, $11^2 = 121$, $27^2 = 729$

IV. For every natural member n, sum of first n odd natural numbers $= n^2$

Example: $1 + 3 + 5 = 3^2 = 9$ = Sum of first 3 odd numbers

V. Pythagorean Triplet

Three natural numbers a, b, c are said to form a Pythagorean Triplet (a, b, c) if

$$a^2 + b^2 = c^2$$

Example: (8, 15, 17) is a Pythagorean triplet as $8^2 + 15^2 = 17^2$.

$64 + 225 = 289$

Example 3: Find the Pythagorean triplet whose smallest number is 12.

Solution: For every natural number, $m > 1$.

$(2m, m^2 - 1, m^2 + 1)$

$$2m = 12 \Rightarrow m = \frac{12}{2} = 6$$

So, the numbers are $2m = 2 \times 6 = 12$

$m^2 - 1 = 6^2 - 1 = 35$

$m^2 + 1 = 6^2 + 1 = 37$

So, required Pythagorean triplet is (12, 35, 37)

Square Root of a Perfect Square by Prime Factorisation Method

1. Resolve the given number into prime factors.
2. Make pairs of similar factors.
3. In product of prime factors, we have to choose one factor out of every pair.

Example 4: Find the square root of 11025.

Solution: $\sqrt{11025}$

$= \sqrt{5 \times 5 \times 3 \times 3 \times 7 \times 7}$

$= 5 \times 3 \times 7$

$= 105$

Example 5: Find the least number that should be multiplied to 360 to make it a perfect square.

Solution: $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$.

To make 360 a perfect square, we must multiply it by $2 \times 5 = 10$

Example 6: Evaluate $\sqrt{6241}$ using long division method.

Solution:

7	$\overline{62}\ \overline{41}$	79.
7	49	
149	1341	
9	1341	
158	××××	

$\sqrt{6241} = 79$

Example 7: By what least number should 7623 be multiplied to get a perfect square.

Solution: $7623 = 3 \times 3 \times 7 \times 11 \times 11$

To make it a perfect square, it must be multiplied by 7.

Multiple Choice Questions

1. What is the value of x if $\sqrt{\frac{2x-1}{3}} = 5$?

   (a) 26 (b) 28 (c) 36 (d) 38

2. What is the greatest number of five digits which is a perfect square?

   (a) 99586 (b) 99856
   (c) 99568 (d) 99865

3. What is the least number which must be subtracted from 6459 to make it a perfect square?

   (a) 56 (b) 58
   (c) 59 (d) None of these

4. By what least number should 384 be multiplied so that the product may be a perfect square?

   (a) 2 (b) 58 (c) 4 (d) 6

5. What is the smallest number of four digits which is a perfect square?

   (a) 1016 (b) 1024 (c) 1036 (d) 1048

6. What is the greatest number of four digits which is a perfect square?

   (a) 9801 (b) 9816 (c) 9824 (d) 9864

7. Find the least number which must be added to 2292 to make it a perfect square.

   (a) 10 (b) 11 (c) 12 (d) 14

8. Find the smallest number by which 557568 must be divided so that it become a perfect square.

   (a) 2 (b) 3 (c) 4 (d) 6

9. Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.

   (a) 3 (b) (c) 9 (d) 11

10. What is the least number which must be added to 6203 to obtain a perfect square ?

    (a) 32 (b) 34
    (c) 36 (d) 38

11. Find the smallest square number that is divisible by each of the number 8, 15 and 20.

    (a) 900 (b) 1600
    (c) 2500 (d) 3600

12. What is the least number that must be added to 1300 so as to get a perfect square?

    (a) 36 (b) 39 (c) 49 (d) 69

13. What is the length of the diagonal of a square whose perimeter is equal to the perimeter of an equilateral triangle of side 4 cm?

    (a) 4. 24 cm (b) 4.04 cm
    (c) 4.14 cm (d) 4.64

14. The product of two positive numbers is $29\frac{31}{49}$ and one of them is three times the other. Find the larger number.

    (a) $\frac{22}{7}$ (b) $\frac{66}{}$ (c) $\frac{37}{7}$ (d) $\frac{68}{7}$

15. The area of a rectangular field whose length is three times its breadth is 348 m^2. What is the perimeter of the field?

    (a) 82.16 m (b) 84.16 m
    (c) 86.16 m (d) None of these

16. Which of the following is not a Pythagorean triplet?

    (a) (3, 4, 5) (b) (6, 8, 10)
    (c) (2, 3, 4) (d) (12, 35, 37)

17. The area of a square field is 60025 m^2. A man cycles along its boundary at 18 km/hour. In how much time will he return to the starting point?

    (a) 166 seconds (b) 176 seconds
    (c) 196 Seconds (d) None of these

18. What is the length of each side of a square whose area is equal to the area of a rectangle of length 13.6 m and breadth 3.4 meters?

    (a) 4.8 m (b) 5.8 m
    (c) 6.8 m (d) 7.8 cm

19. The perimeter of a square field is 76m. What is its area?

    (a) 324 m^2 (b) 289 m^2
    (c) 361 m^2 (d) 329 m^2

20. Which of the following is the square of an even number ?

(a) 729 (b) 324 (c) 441 (d) 625

21. Which of the following is the square of an odd number?

(a) 2209 (b) 1444

(c) 2704 (d) 4096

22. Which of the following is not a perfect square?

(a) 1156 (b) 1764

(c) 1849 (d) 1349

23. What is the cost of erecting a fence around a square field whose area is 9 hectares if fencing costs ₹ 35 per meter?

(a) ₹ 28000 (b) ₹ 32000

(c) ₹ 36000 (d) ₹ 42000

24 In a cinema hall, the number of rows is equal to number of seats in each row. If the capacity of the hall is 2025 what is the number of seats in each row?

(a) 35 (b) 45 (c) 55 (d) 65

25. Which of the following is a Pythagorean triplet?

(a) (2,3,5) (b) (5,7,9)

(c) (6,9,11) (d) (8,15,17).



Cubes and Cube Roots

4

Cube

When a number is multiplied by its square, then the number is said to be cubed, and the product is called the cube of that number.

In general, cube of a natural number 'a' will be $a \times a \times a = a^3$.

A natural number is said to be a perfect cube if it is obtained by multiplying a natural number twice by itself, i.e., a natural number 'η' is a perfect cube if there exists a natural number m such that

$$\eta = m \times m \times m = m^3.$$

Example 1: Is 1728,a perfect cube?

Solution: Here

$$1782 = \underbrace{2\times2\times2} \times \underbrace{2\times2\times2} \times \underbrace{3\times3\times3}$$

In the prime factorisation of 1728, the prime factors can be grouped into triplets of equal factors and no prime factor is left over. Thus, 1728 is a perfect cube.

Example 2: Find the number of cubes of side 5 cm which can be generated by a cuboid of volume 500 cm^3.

Solution: Volume of cube = $5 \times 5 \times 5 = 5^3 = 125\ cm^3$.

Let the number of cubes be x.

$\therefore \quad x \times 125 = 500$

$\Rightarrow \quad x = 4$

$\therefore$ Number of cubes that can be generated by a cuboid of volume 500 $cm^3 = 4$.

Cubes of Negative Integers and Rational Numbers

The cube of a negative integer will be negative. If 'p' is a natural number, then,

$$(-p)^3 = -p^3$$

Example 4: Find the cube of $\left(\frac{-11}{12}\right)$.

Solution: $\left(\frac{-11}{12}\right)^3 = \frac{-1331}{1728}$

Example 5: Evaluate: $\left(\frac{64}{125}\right)^{2/3}$

Solution: Since $\left(\frac{64}{125}\right)^{1/3} = \frac{4}{5}$

$\therefore \quad \left(\frac{64}{125}\right)^{2/3} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

Cube Roots

A natural number m is the cube root of a natural number n, if, $m^3 = n$.

The cube root of a number n is denoted by the symbol $\sqrt[3]{n}$.

Cube Root Using Ones Digit

Observing the unit digits of the number and their cubes, we find that these units digits are either the same or ten's complements. (Remember, complement of 2 in 10 is 8 and vice versa, complement of 3 in 10 is 7 and vice versa, etc.,)

The numbers ending with 0 or 1 or 4 or 5 or 6 or 9 have their respective cubes ending with the same digit.

Example 6: Find the cube root of 9261.

Solution: Since the perfect cube 9261 ends in 1, the cube root must end in 1. Further, it is a 4-digit number. So, its cube root must be a 2-digit number.

Also, $20^3 = 8000$, $30^3 = 27000$

$\therefore$ The cube root of 9261 must be equal to 21.