A challenge for mathematics education: To reconcile commonalities and differences

Un défi pour l'éducation mathématique: Réconcilier le commun et le divers

CIEAEM 54

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l'Étude et L'Amélioration de
l'Enseignement des Mathématiques

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Mathematics Education

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CIEAEM 54

Vilanova i la Geltrú, Spain

Edited by:
Joaquim Giménez, Gail E. FitzSimons, Corinne Hahn

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Presentation

The proposed theme Challenge for Maths Education: To reconcile commonalities with differences, is not an easy topic, but an important reference for future discussions about mathematics education for the 21st century, and a consequence of the suggestions of our Manifesto for the Year 2000, the Year of Mathematics! We are grateful for the positive feedback and support from colleagues all over the world that we already got. We hope you will find the book of the conference proceedings interesting.

It was the first CIEAEM in Catalonia, and we have chosen a village near the Mediterranean sea, in which the Polytechnical University has just celebrated its 75th anniversary and has new residence buildings! The last big Spanish CIEAEM conference was held in Santiago 1978, in memory of Pedro Puig Adam, a Spanish member of the Commission. Twenty four years later, we want to explore with you how to develop possibilities for improving mathematics teaching taking into account the framework of globalisation and internationalisation with its chances and risks. We also included two special sessions on Tertiary Applied Mathematics Education. In one of these, we honour another Catalan mathematician: Luis Antoni Santaló i Sors who died recently. Working most of his life in Argentina, he became famous for his challenging contributions to mathematics education with strong connections to applied mathematics. We invite you to read in the book about many events we had in the Conference: Plenaries, Working groups, Workshops, Special Sessions, and Forum of ideas.

We did our best for organising the Conference according to your needs and wishes, and hope that you enjoyed the beauty of the region. Let us give a special thaks to local organizers, Garraf Association of Teachers of Mathematics, IPC, local Institutions (Vilanova Municipality, Polytechnical University, Barcelona University, Generalitat de Catalunya) who supported financially many activities as Puig Adam exhibition. Thanks to all participants, the international quality class of students, exhibition organizers, groups of teachers doing «street maths» and «mathematics games», Mr Bassegoda with his inspiring Gaudi lecture, and the students of the Doctoral Program of the University of Barcelona who supported us so immensely. With their presence, we might be able to experience exemplarily a sample of a new «young generation» of extraordinary mathematics teachers/researchers, sharing the CIEAEM ideas and principles.

Discussion paper

Document de travail

A challenge for mathematics education: To reconcile commonalities and differences

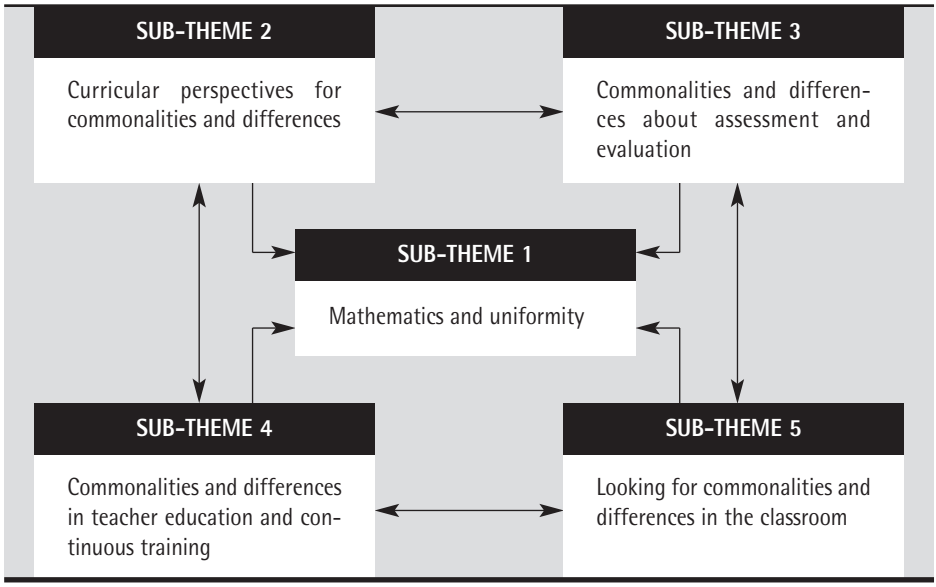
C. Keitel, J. Giménez, C. Hahn, L. Santos, D. Paola

Teaching is now more influenced by policies, curriculum, assessment and teacher education as action processes than in the past. In fact, at the end of 20th century, many educational policies try to impose general standards and formats for all, in a frame of globalisation. It also appears to be the case for mathematics education. Against it, critical issues appear: enculturation is regarded as an opportunity for overcoming western dominance, and ethnomathematics, sociocultural and gender issues are suggested as new lines for research in mathematics education. Many difficulties appear for the mathematics education community because of a new reflection about objectivity in a world dominated by economical and technological resources. Cultural diversity was a first reflection on this issue in the CIEAEM conference at Chichester (Ahmed *et al.*, 2001), but other aspects are involved in the actual analysis of the influences of a globalised society. It leads to a problem of commonalities and differences in a wider sense than the cultural one. Knowledge societies, knowledge systems (Keitel, 2001) and traditions must be reconciled as outcomes of a cultural practice and inputs to learning.

After several years of «mathematics for all» interdependency and freedom are now elements to introduce in the framework for Mathematics Education. More than ever before, people need to discuss sociocultural interchanges and original issues revealed to be important to overcome universalist assumptions. Interpersonal comparisons and global advantages need a set of heterogeneous components (Sem, 1999). In this framework, research and practice on mathematics education must reflect on these new perspectives, looking at the the risks of uniformity, a sense of conservation, and the promotion of human being.

A new challenge is proposed: How to reconcile commonalities and differences in managing mathematics information: reflecting curricular issues, promoting new teacher training, and reflecting on classroom practice. What does mathematics for all mean now? Is it possible to talk about mathematics and uniformity? To think about it means to ask ourselves about how to overcome eurocentrism, to promote cooperation and to think about the role of applied mathematics and technological tools. But this is strongly related to curricular perspectives, new assessment procedures, teacher education models and classroom research. Therefore, we need to recognise new sociocultural perspectives, cognitive science results and neuroscience research perspectives about commonalities in knowledge system studies. Let's see it as an inter-related network as sub-themes for our conference.

The relations between the sub-themes



Sub-theme 1: Mathematics and uniformity

Mathematics has been commonly understood as an effective system for symbolic actions, as a tool for critical interpretation of the real world, or as a fascinating activity of human beings (Arzarello, 2000). Curricular issues, from this perspective, are a meeting point for the development of diverse theoretical and practical competences. Educational policies aiming at standardisation, however, could turn into negative impositions. Are there commonalities or standards that we must conserve when we refuse global uniformity?

How might we reconcile the eventual opposition of economic demands and social or pedagogical needs? Should mathematics education be considered as a part of general education or rather from the perspective of professional training for the few? In some countries, mathematics education is no longer considered as necessary and imperative as before, and even less time is devoted for specialized or advanced training in the discipline. Obviously, the loss of certainty in mathematics is misunderstood as a loss of the importance of mathematical knowledge (Ernest, 1998) Who designs common or uniform competences to be provided by mathematics education and why? If we recognise the fact that mathematics, by the social and technological development of our societies, has become more implicit and invisible although more widespread and uniform as social and material technology, how is this reflected in mathematics learning and teaching? If we want common educational outcomes, does this mean that we have to start with common input? (Willis, 1998) What are the roles of researchers, schools, and teachers in defining uniform or diverse competences? (CIEAEM 2000).

KEY QUESTIONS

- What are the political, social, and cognitive implications of uniformity in mathematics education?
- Is formalism more than an effective mathematical framework? Are there implicit and explicit models in teaching mathematics?
- What is the role of applied mathematics in searching for commonalities and relationships?
- What are the differences that must be conserved in mathematics education?
- What experiences do we have in assuring equitable partnerships and sharing experiences among different countries? How might we design mathematics education research to be participative - involving the community, students, teachers, and policy makers, etc. as well as academics?
- What might be the role of values, legitimation, representation forms, performative knowledge, and philosophical perspectives in relating commonalities and differences?

Sub-theme 2: Curricular perspectives for commonalities and differences

In a world in which ideas, commodities, people and information flow more and more easily, the curriculum is asked to satisfy international standards in order to facilitate such circulation. And suddenly there is also a drive towards redefinition and new standardisation of contents, in order to match the impositions of comparative assessment studies.

Questions of standard become particularly important and urgent when the agenda asks for defining a mathematics curriculum for vocational training and tertiary education for non-mathematicians. Should we propose the same common general mathematical culture for everybody? Certainly, we know that to propose the same mathematics for all tends to impose the dominant culture because mathematics is not neutral, culturally speaking (Saxe, 1991). But, is it possible to define the mathematics that will be needed for all, as a package of mathematical knowledges for a world citizen? Which international curricula should we experiment with in order to build these mathematical packages? Many research studies show the difficulties of transferring school mathematics knowledge into an enterprise (Noss and Hoyles, 1996; Hahn, 2000). We also know that it is important that learners generate problems by themselves (Boaler, 1998). Is there a role for learners as curriculum builders? Should the curriculum just show examples or be defined by means of basic activities for the classroom? What is known from research about social influences over curriculum, and what research do we need for coming years? It is also necessary to explore how collaborative work offers the opportunity not only for confronting ideas, but also cultures and experiences. In addition, exploration is needed into how technology can be a bridge among cultures, an instrument for individualizing training processes, and a tool for overcoming students' epistemological anxieties in the face of concepts whose legitimacy is questioned.

KEY QUESTIONS

- Is there a package which contains the mathematics that each and everybody will need to master the problems of the 21st century, irrespective of the cultural environment? Is it possible and desirable to construct a universal curriculum?
- What are the methods which will allow us to reconcile globalisation and cultural diversity?
- How can we put these methods to work in the diverse classroom?
- Are technologies a vector for the integration of cultural diversity in a global society or, on the contrary, do they represent a danger of imposing the dominant culture?

Sub-theme 3: Commonalities and differences about assessment and evaluation

As in many other areas in education, the conception of assessment has been changing over time. In the 1950s, to assess the students' mathematics performance meant to «measure» their knowing. Associated with the idea of measurement, several aims have to be attained, such as the objectivity of the evaluators and the instruments. Assessment is viewed as a task for the expert, as an expert judgment (Hadji, 1989). An immediate consequence was the tendency to focus only on the aspects of mathematics that could be measured. The normative nature of the assessment is based on the framework that every student is equal at the beginning. We assess what a certain student knows in comparison with what others know (Cardinet, 1988). We have, as result of this process, an absolute hierarchy in «bad» and «good» students (Perrenoud, 1989).

More recently, different frameworks have been established. Considering assessment as an social activity, the diversity is a fundamental aspect to be considered. Different forms of learning, gender differences, social and cultural contexts, determine diverse ways and times for the processes of teaching, learning and assessment. Social justice and equity have new meanings (Keitel, 1998). Assessment as a communication process is an inseparable part of the teaching process, contributing to the making and adapting of teachers' decisions, and to the learning process, regulating the progress of each student.

In the assessment process teachers and students have both a central role. The requirement for a transparent process (NCTM, 1995) emerges not only due to an ethical exigency but also as indispensable to develop auto-regulation (Nunziati, 1988). The complexities of the new demands are high. The teacher is asked, for example, to develop a differential teaching (not an individualized one), to explicate the assessment's criteria, to analyse the errors (to interpret and formulate possible explanations), to give adequate feed-back and to develop adequate forms of data collection. The student has to develop his or her ability to understand and reflect on their mathematics competence. No longer is it important to learn in mathematics is what is easily measured, but instead what may contribute to all students having mathematical power (CIEAEM 2000).

At the same time, concerns with the quality of teaching and learning mathematics, sometimes associated with economically inspired approaches to education, have pro-

duced, among other responses, the development and conduct of international comparative studies, such as TIMSS. These studies may contribute to a more common discourse or even to a more effective communication all over the world. However, when there results are read in a linear way, without attention to the diversity of the contexts, assessment can again become rapidly just a measurement process, associated with the reporting of a ranking of countries and regressing the main purposes of mathematics education, instead of contributing to the improvement of each educational system.

KEY QUESTIONS

- How can teachers develop a culture of assessment as an integral part of the teaching and learning process? In particular, how can teachers promote formative interaction? How does this contribute to the development of auto-assessment to regulate the process?
- What strategies have to be implemented to guarantee a coherent, equitable and just process in a community of social and cultural diversity? In particular, what new forms of assessment are required?
- How could international comparisons improve and develop productive ways for teaching-learning in specific cultural and social settings? How could the establishment of international standards be compatible with cultural diversity?
- How can communities with different political systems, social conditions and cultural traditions search for productive ways to co-operate and learn from each other in assessment questions?

Sub-theme 4: Commonalities and differences in teacher education and continuous training

The Professional Standards (NCTM 1999) for mathematics education can be considered as an important framework for teacher training practices and at the same time as a symbol for a typical approach proclaiming to be research-based and valid for a globalised world. Problem solving activities related to real situations, and learning by reflective practice were important issues - although not really new - which offered a common perspective for debates in this field. Several theoretical concepts and practical frameworks have been introduced, such as: Professional knowledge systems, anthropological perspectives, social-cultural and critical paradigms and related interventions etc. What are the commonalities between them? How can they help teachers to improve their professional growth in a way that accepts future cultural and social differences? In many countries the increasing importance of mathematics education for future primary teachers has been disucssed. How did the framework react to their needs?

Mathematics teachers often interpret their job as being obliged to impose institutional demands driven by institutional knowledges on the students. To be open to new forms of understanding about learning processes as well as the mathematics to be learnt and entering new discourses in the professional community might be hindered by already acquired intuitive models that act as obstacles or only refer to very formal prin-

ciples. Therefore any intervention model in teacher education must allow the teacher to interpret the new conceptionalisation of «teacher», «student» and «institution» in such a way that the student can positively react and reward teacher actions by self-guided learning (Cabral, 2001). Traditional methodologies must change because they fail in developing students' independent learning habits and knowledge acquisition. What role could be played by individuals' and teacher's beliefs?

Lave emphasized that teacher further and continuing education has to focus on a reflection of practice from a critical cognitive position (Lave, 1988). There were searches for homogeneity in guiding principles on pedagogical practices, but studies on interaction and classroom communication revealed problems of teaching practice, and that heterogeneity offers better means for collaborative action and provides mutual benefit for teachers and researchers as well as an enlightenend practice (Porfirio and Abrantes, 1999). Research studies with a socio-cultural approach on classroom culture, analysing the argumentative discourses, provide better explanations for the evolution of knowledge. In assessment procedures, for instance, criteria for validity use, formal resources as a justification, while other studies call for collaborative verification as validity and reliability in which the complicity of a team plays a major role (Hanna, 1989).

KEY QUESTIONS

- How can we involve teacher trainers, teachers and training institutions for openness to complexity? What experiences do you have in this field? How can we prepare teachers for coping better with differences?
- Preparing for complexity and change is another challenge for teacher training: Does this mean that new subjects should be introduced and that contextualised discussion is needed to re-interpret school practices from teacher writings?
- Which experiences of collaborative training can help, especially, young teachers in their first professional experiences?
- Is there a need for revisting action research as the only possible continuous training?
- What is the role of technological issues that help to preserve differences, accepting commonalities?

Sub-theme 5: Looking for commonalities and differences in the classroom

Mathematicians, teachers and mathematics education researchers have always felt the need to identify common features in their studies. They have been trying to single out the peculiar characteristics of mathematical thinking which could help to make sense of the many different cultural perspectives and the various practices, and consequently to characterise mathematics as a subject.

The Bourbakist manifesto and the different answers provided by logicians, intuitionists and formalists to the problem of the foundations of mathematics are examples of explicit attempts towards this aim. All these persepctives, although different in approach

and intention, consider mathematics as a corpus of knowledge independent from the physical and biological nature of human beings: mathematics exists and is written somewhere or it is to be written. In any case, what is discovered or written does not depend on the physical and biological characteristics of human beings. To put it in other words, according to these programs, mathematics is a product of the Cartesian mind.

From a socio-cultural and socio-political perspective, what is the same and what is different is to be explicated by critically analysing, globally, the characteristics of the cultural production of the society, and locally, the socio-cultural conditions of the classroom.

From an enactivist approach in cognitive science (Petitot *et al.*,1998) it is suggested to tackle the problem of what is the same and what is different from another perspective. Petitot states that our mind is deeply embodied, that thinking processes are mostly unconscious, and that abstract concepts, as for example mathematics concepts, are metaphorical. George Lakoff and Rafael Núñez (Núñez, 2000; Lakoff, G. and Núñez, R., 2000) argue in a similar line:

> *The question of the existence of a Platonic mathematics cannot be addressed scientifically. At best it can be a matter of faith [...] As with the conceptualization of God, all that is possible for human beings is an understanding of mathematics in terms of what the human brain and mind afford. [...] Therefore, mathematics as we know it and teach it can only be humanly created and humanly conceptualized mathematics.*

Taking into account these ideas means changing the perspective from which the problem of what is the same and what is different in the classroom can be investigated or analysed: our physical and biological nature, our way of experiencing the world, our models of abstract knowledge, and mathematical knowledge in particular.

Mathematical knowledge can be considered as formed, structured and communicated by means of metaphors, which are eventually linked to the way our sensorimotor apparatus interacts with the world. According to this perspective, natural common foundations of knowledge in general, and mathematical knowledge in particular, exist beyond social and cultural differences: they are the physical and biological nature of human beings (compare also the six universal mathematical activities stated by Bishop, 1989). Even if different cultural and social conditions contribute to the development of apparently very different conceptual systems, almost all human beings interact with the world through sight, taste, touch, sense of smell, and they process the data gathered from the outside world by means of a similar sensorimotor apparatus.

KEY QUESTIONS

- What might an enactivist approach contribute to the mathematics classroom? What does enactivism contribute to research in mathematical classroom?
- If abstract concepts are metaphorical, as the enactivism theory suggests, what are the metaphors used in the production, systematisation and communication of mathematical thinking?
- How is mathematical knowledge structured in the subjects? How is mathematical thinking activated?

- Is there a special role for a situated/embodied cognition in teacher training processes? Is this model sufficient for interpreting continuing and pre-service teacher training in mathematics education? What is the role for virtual environments in teacher education? Are teacher belief systems based just on a cultural reality? Are they used only in specific chosen contexts?
- New technologies provide the possibility of making more experiences. Can this help the development of more powerful metaphors aimed at constructing, organising and communicating mathematical thinking?
- Access to technologies may vary from school to school. Is there a risk of amplifying the already existing differences among classrooms and within classrooms, and consequently producing unnecessary social and cultural differences? What is the responsibility of teachers in this process?

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Un défi pour l'éducation mathématique: Réconcilier le commun et le divers

C. Keitel, J. Giménez, C. Hahn, L. Santos, D. Paola

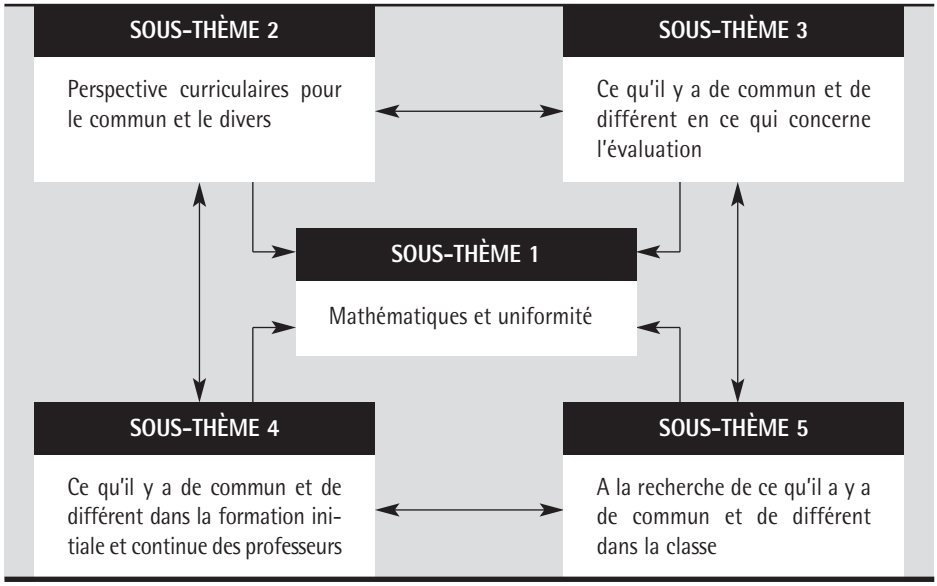
Aujourd'hui plus que par le passé, l'enseignement est influencé par la politique, les programmes, les évaluations et la formation des enseignants comme par les processus d'action. En fait, à la fin de 20ème siècle, plusieurs politiques éducatives ont essayé d'imposer des standards universels, dans un cadre de globalisation. Ce fut aussi le cas pour l'enseignement des mathématiques. Mais des critiques ont été faites : la mise en perspective culturelle (appelée «enculturation» en anglais) est regardé comme un moyen de lutter contre la domination occidentale, et l'ethno-mathématiques, les aspects socioculturel et féministe ont été proposés comme de nouvelles pistes pour la recherche sur l'éducation mathématique. La communauté de l'enseignement des mathématiques a rencontré de nombreuses difficultés en raison de la nouveauté de cette réflexion sur l'objectivité dans un monde dominé par les ressources économiques et technologiques. La diversité culturelle a été un premier thème de cette réflexion au cours de la conférence CIEAEM à Chichester mais d'autres aspects sont à considérer dans cette analyse des effets d'une société globalisée. Cela conduit au problème des points communs et des différences en un sens plus large que seulement culturel. Les systèmes de connaissances (Keitel, 2001) et les traditions doivent être réconciliés en considérant leurs rôles complémentaires comme le résultat d'une pratique culturelle ou d'un point de départ des apprentissages.

Après plusieurs années de «mathématiques pour tous», l'interdépendance et la liberté sont maintenant des éléments à introduire dans le cadre de l'éducation mathématiques. Plus qu'auparavant, il faut discuter des échanges socioculturels et des axes de réflexion originaux qui se révèleront être importants pour surmonter les hypothèses universalistes.

Les comparaisons interpersonnelles et les avantages globaux n'ont pas les mêmes systèmes de composantes (Sem, 1999). Dans ce schéma, recherche et pratique doivent réfléchir à ces nouvelles perspectives en tenant compte des risques d'uniformité, de ce qui doit être conservé, et de la promotion des êtres humains.

Un nouveau défi nous est proposé : comment réconcilier ce qu'il y a de commun et ce qui est différent dans la gestion de l'information mathématique : réfléchir aux programmes, proposer une nouvelle formation pour les enseignants et s'interroger sur les pratiques dans la classe. Que signifient aujourd'hui mathématiques pour tous ? Est-il possible de parler de mathématiques et uniformité ? S'interroger sur ce point signifie se demander comment dépasser l'eurocentrisme, promouvoir la coopération et réfléchir au rôle des mathématiques appliquées et des outils technologiques. Mais ceci est fortement lié aux perspectives «curriculaires», aux nouvelles méthodes d'évaluation, aux conceptions de formation des enseignants et à la recherche dans la classe. Nous devons donc envisager de nouvelles perspectives socioculturelles, les apports des sciences cognitives et les perspectives des recherches en neurosciences sur les points communs dans les études sur les systèmes de connaissances. Considérons ainsi ce réseau interconnecté de sous-thèmes pour notre conférence.

Les relations entre les sous-thèmes



Sous-thème 1 : Mathématiques et uniformité

Les mathématiques sont généralement considérées comme un système efficace d'actions symboliques, comme un outil pour une interprétation critique du monde réel ou comme une fascinante activité humaine. La réflexion sur les programmes, dans cette perspective, est le point de rencontre pour le développement de diverses compétences théoriques et pratiques. Les politiques éducatives visant à standardiser, cependant peuvent se révéler négatives. Existe-t-il des standards que nous devons conserver lorsque nous refusons l'uniformité globale ?

Comment sommes nous susceptibles de réconcilier l'éventuelle opposition entre les demandes économiques et les besoins sociaux et pédagogiques ? Est-ce que les mathématiques doivent être considérées comme une partie d'un enseignement destiné à tous ou plutôt comme une formation professionnelle destinée à un petit nombre de personnes ? Dans certains pays, la formation mathématique n'est plus considérée comme indispensable comme c'était le cas auparavant, et encore moins de temps est alloué à la formation spécialisée ou avancée dans cette discipline. Apparemment la perte de certitude en mathématiques est comprise à tort comme une perte d'importance des savoirs mathématiques. Qui définit les compétences communes à construire par la formation mathématique et pourquoi ? Si nous reconnaissons que les mathématiques, compte tenu du développement social et technologique de nos sociétés sont devenues plus implicites et invisibles bien que plus répandues et uniformes comme matériau social et technologique, comment ceci est-il reflété dans l'enseignement et l'apprentissage des mathématiques. Si nous voulons aboutir à un même résultat pour tous, cela signifie-t-il que nous devons partir des mêmes bases ? Quels sont les rôles respectifs des chercheurs, écoles et enseignants dans la définition de compétences communes ou distinctes ?

QUESTIONS-CLÉS

- Quelles sont les implications politiques, sociales et cognitives de l'uniformité dans l'éducation mathématique ?
- Est-ce que le formalisme est plus qu'un cadre mathématique efficace ? Existe-t-il des modèles implicites et explicites pour l'enseignement des mathématiques ?
- Quel est le rôle des mathématiques appliquées dans la recherche sur les points communs et les relations ?
- Quelles différences doit-on conserver dans l'enseignement des mathématiques ?
- Quelle expérience a-t-on pour mettre en place un partenariat équitable et un partage d'expérience entre les différents pays ? Comment définir une recherche en mathématique participante qui implique la communauté, les étudiants, les enseignants et les décideurs etc ; aussi bien que les institutions?
- Quel pourrait être le rôle joué par les valeurs, la légitimation, les formes de représentation, les performances cognitives et les perspectives philosophiques pour lier ce qu'il y a de commun et ce qui est différent ?

Sous-thème 2 : Perspective curriculaires pour le commun et le divers

Dans un monde où les idées, l'information, les biens et les personnes circulent de plus en plus facilement, il est demandé aux programmes scolaires de satisfaire à des standards internationaux afin de faciliter cette circulation. Cela conduit souvent à une redéfinition des contenus et à une nouvelle standardisation afin de répondre aux exigences des évaluations comparatives.

Les questions de standard deviennent particulièrement importantes et urgentes lorsqu'il s'agit de définir un programme pour l'enseignement professionnel et une formation supérieure pour non mathématiciens. Doit-on proposer à tous la même culture mathématique générale ? En effet on sait que proposer les mêmes mathématiques à tout le monde conduit le plus souvent à imposer la culture dominante car les mathématiques ne sont pas neutres culturellement (Saxe, 1991). Mais est-il possible de définir les mathématiques qu'il faudra enseigner à tous, le «pack» de savoirs mathématiques du citoyen du monde ? Quel programme international peut-on expérimenter dans le but de construire ce pack mathématiques ? De nombreuses recherches ont montré que les élèves avaient souvent du mal à transférer en entreprise les savoirs mathématiques appris à l'école (Noss et Hoyles, 1996; Hahn, 2000). Nous savons qu'il est important que les apprenants génèrent leurs propres problèmes (Boaler, 1998). Les apprenants ont-ils un rôle à jouer dans la construction des programmes ? Le programme doit-il simplement montrer des exemples ou doit-il être défini par le biais d'activités fondamentales pour la classe ? Quel est l'apport des recherches sur les influences sociales sur les programmes et de quelles recherches avons-nous besoin dans les années à venir ? Il faut aussi explorer les possibilités offertes par l'apprentissage coopératif, non seulement de confronter les idées mais aussi les cultures et les expériences. Il conviendrait également s'interroger sur la manière dont la technologie pourrait jouer à la fois le rôle de pont

entre les cultures, un outil d'individualisation des formations et un moyen de lutter contre l'anxiété épistémologique que ressentent certains élèves face à des concepts dont la légitimité est mise en doute (Wilenski, 1997).

QUESTIONS-CLÉS

- Existe-t-il un corpus mathématique nécessaire à tous les êtres humains pour maîtriser les problèmes du XXIème siècle, quelque soit leur environnement culturel ? Peut-on, doit-on construire un curriculum universel ?
- Quelles sont les méthodes qui permettront de concilier globalisation et diversité culturelle ?
- Comment mettre ces méthodes en œuvre dans la classe ?
- Les technologies sont-elles un vecteur d'intégration de la diversité culturelle dans une société globale ou au contraire représentent-elles un danger en imposant la culture dominante ?

Sous-thème 3 : Ce qu'il y a de commun et de différent en ce qui concerne l'évaluation

Comme dans d'autres domaines de la formation, la conception de l'évaluation a changé au cours du temps. Dans les années cinquante, évaluer les performances des étudiants en mathématiques signifiait mesurer leurs connaissances. Associés à l'idée de mesure, plusieurs objectifs devaient être atteints comme l'objectivité des évaluateurs et de leurs instruments. L'évaluation est vue comme une entreprise pour les experts. Une conséquence immédiate a été la tendance à se focaliser seulement sur les aspects des mathématiques qui pouvaient être mesurés. L'aspect normatif de l'évaluation est basé sur le postulat selon lequel tous les étudiants sont égaux au départ. Nous évaluons ce qu'un étudiant sait en comparaison avec ce que les autres savent. Nous obtenons à l'arrivée une hiérarchie absolue de «bons» et «mauvais» élèves.

Plus récemment, plusieurs cadres de référence ont été établis. Considérant l'évaluation comme une activité sociale, la diversité est un aspect qu'il est fondamental de prendre en compte. Les différentes formes d'apprentissage, les différences liées au sexe, les contextes socioculturels déterminent des moyens différents pour les processus d'enseignement, d'apprentissage et d'évaluation. La justice sociale et l'équité ont de nouvelles significations. L'évaluation comme un processus de communication est inséparable du processus d'enseignement, contribuant à faire et adapter les décisions des enseignants et du processus d'apprentissage permettant de réguler les progrès de chaque étudiant.

Au cours du processus d'évaluation, les enseignants et les élèves ont un rôle central. Une demande de transparence émerge, non seulement en raison d'exigences éthiques mais aussi parce que cette transparence est indispensable pour développer l'auto-régulation. Les complexités de cette nouvelle demande sont élevées. Par exemple, on demande au professeur de développer un enseignement différencié (et non pas individualisé) d'expliquer les critères d'évaluation, d'analyser les erreurs (pour interpréter et formuler des explications possibles), de donner un feed-back adéquat et de développer des formes adéquates de collecte de données. L'élève doit développer sa capacité à comprendre et à réfléchir sur sa compétence mathématique. Il n'est plus important

d'apprendre en mathématiques ce qui est facilement mesurable mais ce qui peut contribuer à donner aux élèves le pouvoir mathématique.

Au même moment, le soucis de qualité de l'enseignement et l'apprentissage des mathématiques, parfois associé avec une approche économique de la formation ont produit, ont produit, parmi d'autres réponses, le développement et la conduite d'études comparatives internationales, comme TIMSS. Ces études peuvent contribuer à mettre en place un discours commun voire une communication plus effective partout dans le monde. Quoiqu'il en soit, lorsque les résultats sont lus de manière linéaire sans porter attention à la diversité des contextes l'évaluation peut devenir rapidement juste un processus de mesure associé à un classement des pays contribuant à une régression des principaux objectifs de l'éducation mathématique au lieu de contribuer à améliorer chaque système éducatif.

QUESTIONS-CLÉS

- Comment les enseignants peuvent-ils développer une culture de l'évaluation comme une part intégrante du processus d'enseignement et d'apprentissage ? En particulier comment les enseignants peuvent-ils promouvoir l'interaction formative ? Comment cela contribue-t-il au développement de l'auto-évaluation pour réguler le processus ?
- Quelles stratégies doivent être mises en place pour garantir un cohérent, équitable et juste processus dans une communauté diverse sur le plan social et culturel ? En particulier, quelles nouvelles formes d'évaluation faut-il développer ? Comment considérer l'étudiant à la fois comme un individu et comme membre d'un groupe ?
- Comment les comparaisons internationales peuvent-elles améliorer et développer la relation enseignement / apprentissage dans des environnements particuliers sur le plan culturel et social? Comment la mise en place de standards internationaux est-elle compatible avec la diversité culturelle ?
- Comment des communautés dotées de systèmes politiques, de conditions sociales et de traditions culturelles différents peuvent-elles réfléchir à des moyens productifs de coopérer et d'apprendre les unes des autres sur ces questions d'évaluations ?

Sous-thème 4 : Ce qu'il y a de commun et de différent dans la formation initiale et continue des professeurs

Les standards professionnels pour l'éducation mathématique peuvent être considérés comme un cadre important pour les pratiques de formation des enseignants et en même temps comme emblématiques d'une approche typique se proclamant basée sur la recherche et valide pour un monde globalisé. Les activités de résolution de problèmes liées au monde réel et d'apprentissage à partir de pratiques réflexives ont été des aspects importants -bien que pas vraiment nouveaux- offrant une perspective commune de débats à ce sujet. Plusieurs concepts théoriques et des schémas pratiques ont été introduits, tels que : les systèmes de savoirs professionnels, les perspectives anthropo-

logiques, les paradigmes socio-culturels et critiques et les interventions correspondantes etc. Quels sont les points communs entre eux ? Comment pourraient-ils aider les enseignants à améliorer leur évolution professionnelle pour s'adapter aux différences sociales et culturelles à venir ? Dans de nombreux pays on a contesté l'importance grandissante accordée à l'éducation mathématique pour les futurs instituteurs. Comment ce schéma s'est-il adapté à leurs besoins ?

Les professeurs de mathématiques considèrent souvent que leur métier consiste à enseigner des savoirs imposés par les institutions imposer aux étudiants les demandes institutionnelles conduites par les savoirs institutionnels. Etre ouverts aux nouvelles formes de compréhension des processus d'apprentissage aussi bien qu'aux mathématiques à faire acquérir et entrer dans de nouveaux discours au sein de la communauté professionnelle est rendu difficile par les modèles intuitifs précédemment acquis qui y font obstacle ou tout au moins renvoient à des principes très formels. Pour ces raisons, tout modèle d'intervention concernant la formation des maîtres doit permettre une nouvelle conceptualisation de la relation professeur, étudiant et institution telle que l'étudiant puisse réagir positivement aux actions du professeur par apprentissage autoguidé. Les méthodologies traditionnelles doivent changer parce qu'elles échouent à développer chez les étudiants des habitudes d'apprentissage et d'acquisition du savoir autonomes. Quel rôle pourraient jouer les croyances des individus et des professeurs ?

Lave a insisté sur le fait que la formation continue des professeurs doit se centrer sur une réflexion sur la pratique d'un point de vue cognitif critique. Il y a eu des recherches pour homogénéiser les principes guidant les pratiques pédagogiques mais les études sur l'interaction et la communication dans la classe ont révélé des problèmes de pratique enseignante, et que cette hétérogénéité est meilleure pour du travail en commun et apportent un bénéfice mutuel pour les chercheurs et les enseignants ainsi qu'une pratique allégée. Les recherches avec une approche socioculturelle sur la culture de la classe analysant les discours argumentatifs, apportent de meilleures explications concernant l'évolution du savoir. Dans les procédures d'évaluation par exemple les critères de validité usent des ressources formelles comme justification pendant que d'autres études se réfèrent à la vérification en commun de la validité et de la pertinence dans laquelle la complicité d'équipe joue un rôle majeur.

QUESTIONS-CLÉS

- Comment pouvons nous entraîner les formateurs de formateurs, les enseignants et les instituts de formation à s'ouvrir à la complexité ? Quelles expériences avons nous dans ce champ? Comment pouvons-nous préparer les enseignants à mieux profiter des différences ?
- Préparer à la complexité et au changement est un autre défi pour la formation des enseignants : est ce que cela signifie qu'il faut introduire de nouveaux sujets et qu'il est nécessaire d'avoir recours à la discussion contextualisée pour ré-interpréter les pratiques scolaires à partir des écrits des enseignants ?
- Quelles expériences de formation collaborative peuvent aider, en particulier les jeunes professeurs lors de leur première expérience professionnelle ?
- A-t-on besoin de revisiter la recherche action comme la seule forme de formation continue ?
- Quel est le rôle des expériences technologiques qui aident à préserver les différences en acceptant ce qu'il y a de commun ?

Sous-thème 5 : A la recherche de ce qu'il y a de commun et de différent dans la classe

Les mathématiciens, les enseignants et les chercheurs en éducation mathématique ont toujours ressenti le besoin d'identifier des traits communs à leurs études. Ils essayent d'identifier les caractéristiques de la pensée mathématique qui pourraient aider à donner du sens aux différentes perspectives culturelles et aux pratiques diverses, et par conséquent caractériser les mathématiques comme un sujet.

Le courant Bourbaki et les différentes réponses apportées par les logiciens, intuitionnistes et formalistes au problème des fondements de la mathématique sont des exemples d'essais explicites d'atteindre ce but. Toutes ces perspectives, bien que différentes dans leurs approches et leurs intentions, considèrent les mathématiques comme un corpus de savoirs indépendants de la nature physique et biologique des êtres humains. Pour dire cela autrement, selon ces programmes, les mathématiques sont le produit de l'esprit cartésien.

D'un point de vue socioculturel et socio-politique, ce qui est identique et ce qui est différent doit être expliqué par l'analyse critique, globalement, des caractéristiques de la production culturelle de la société et localement, par les conditions socioculturelles de la classe.

Dans une approche «énactive» en sciences cognitives, il est suggéré d'aborder le problème de ce qui est identique et ce qui est différent d'un autre point de vue. Petitot postule que notre esprit est profondément intégré dans notre corps, que les processus de pensée sont le plus souvent inconscients, et que les concepts abstraits, comme par exemple les concepts mathématiques sont d'ordre métaphorique. Georges Lakoff et Nuñez affirment, dans le même ordre d'idée, que :

> *The question of the existence of a Platonic mathematics cannot be addressed scientifically. At best it can be a matter of faith [...] As with the conceptualization of God, all that is possible for human beings is an understanding of mathematics in terms of what the human brain and mind afford. [...] Therefore, mathematics as we know it and teach it can only be humanly created and humanly conceptualized mathematics.*

Prendre en compte ces idées signifie changer la perspective selon laquelle le problème de ce qui est commun et ce qui est différent peut être exploré et analysé. : notre nature physique et biologique, notre façon d'appréhender le monde, nos modèles de savoir abstrait et le savoir mathématique en particulier.

Le savoir mathématique peut être considéré comme formé, structuré, et communiqué par le biais de métaphores, qui sont liées à la manière dont notre système sensori-moteur interagit avec le monde. Selon cette perspective, les fondements naturels communs du savoir en général et du savoir mathématique en particulier, existe au de là des différences sociales et culturelles : elles sont la nature physique et biologique des êtres humains (à comparer aussi aux six activités mathématiques universelles mises en évidences par Bishop). Même si les conditions culturelles et sociales contribuent au développement de systèmes conceptuels apparemment très différents, presque tous les êtres humains interagissent avec le monde au travers de la vue, du toucher, du goût, de

l'odorat et ils analysent les données ramenées du monde extérieur par le biais du même appareil sensori-moteur.

QUESTIONS-CLÉS

- Qu'est-ce qu'une approche de type «enactiviste» pourrait apporter à la classe de mathématiques ? Qu'est-ce que l'énactivisme peut apporter à la recherche sur la classe de mathématique ?
- Si les concepts abstraits sont métaphoriques, comme le suggère la théorie énactiviste, quelles sont les métaphores utilisées dans la production, la systématisation et la communication sur la pensée mathématique ?
- Comment le savoir mathématique se structure-t-il dans les sujets ? comment est-ce que le savoir mathématique est activé ?
- Existe-t-il un rôle spécial pour la cognition située / incorporée dans les processus de formation des enseignants ? Ce modèle peut-il suffire pour interpréter les formations initiales et continues en éducation mathématique Quel est le rôle des environnements virtuels dans la formation des enseignants ? Est-ce que les systèmes de croyance des enseignants sont seulement basés sur une réalité culturelle ? Sont-ils utilisés seulement dans certains contextes spécifiques ?
- Les nouvelles technologies apportent la possibilité de faire encore plus d'expériences. Est-ce que cela peut aider au développement de métaphores plus puissantes visant à construire, organiser et communiquer la pensée mathématique ?
- L'accès aux technologies varie d'une école à l'autre. Y a t-il un risque d'amplifier les différences existantes entre les classes et à l'intérieur de la classe et en conséquence de produire des différences sociales et culturelles qui ne seraient pas nécessaires. Quelle est la responsabilité des enseignants dans ce processus ?

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Part I / Partie I
Plennary sessions / Sessions plénières

1

Too much is not enough: Teaching maths through useful applications with local and global perspectives

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Abstract
According to a situation of change, a new global perspective for preparing citizens in mathematics education is revisted. The main role for applications in a globalized perspective is discussed and analyzed.

Résumé
En considérant une situation de changement, nous revisiterons l'éducation mathématique des citoyens selon une nouvelle perspective globale. Nous discuterons et analyserons le rôle fondamental des applications dans cette perspective.

An initial thought...

Education is for the majority of people the only key to their future. Mathematics is in this sense a major tool to allow this key to turn in the keyhole of our society.

When learning mathematics was a minority activity, mathematics could be «universal». Now that we want to reach «all» people «everywhere» mathematics needs to play a subtle game of giving as many opportunities as possible to everyone.

The world has changed; the perspectives, values and aims of our students have changed... so it is time for us to make some crucial changes in mathematics education.

Local citizens with global perspectives

At compulsory level we prepare future citizens in a very specific social context. Our maths teaching may benefit from local characteristics and it is our goal to prepare students to be critical citizens and good professionals in their contexts.

On the other hand the increasing phenomenon of globalization needs to be faced and clarified, since the term «globalization» hides and mixes various meanings in polit-

ical terms, in economic settings, in immigration processes, etc. From our perspective as mathematics educators we need to take into account the following main aspects of globalization:

- Common global problems.
- Global information networks.
- Global markets.
- Global travelling.
- Global immigration of individuals.

Our chief concern in this lecture is to see how maths teaching through applications can be sensitive to both local and global perspectives.

If learning is the question... quantitative literacy is the answer

In the International Life Skills Survey (ILSS, 2000) we can find a very clear and comprehensive definition of *quantitative literacy*:

> *An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.*

Recently, Lynn Arthur Steen (see Steen, 2001, p. 8-9) identified a very complete list of elements which are characteristic components of *quantitative literacy*:

- Confidence with Mathematics.
- Cultural Appreciation.
- Interpreting Data.
- Logical Thinking.
- Making Decisions.
- Mathematics in context.
- Number sense.
- Practical Skills.
- Symbol Sense.
- Prerequisite knowledge.

If we accept as a starting point that compulsory education needs to focus on quantitative literacy then it is easy to see that *dealing with applications and modelling skills* is an ideal way to approach our teaching/learning goals.

If quantitative literacy is the answer... applications are the solution

Throughout this lecture we will defend the idea that realistic teaching is an appropriate way towards quantitative literacy training. Motivating by means of useful

applications, working on modelling, solving problems... this is the way we consider ideal for our mathematics teaching at all levels.

An important consequence of teaching «via applications» is that the classical way to deliver lectures needs to be changed. Teaching with applications today means stopping the «talk and chalk» method, stopping using an old textbook and offering a very lively guiding program, based upon various information sources, with revised teaching notes and opening new windows to appreciate the context of the students and their creativity as individuals and as a group.

So we will focus only on *real applications*, based on *real data*, on *objects* and *instruments*, on *everyday* situations, on *frequent* or *recent events*. Along these lines, the American educational projects «Modelling our world» and «Maths in context» are very fine examples of how textbooks can facilitate the realistic approach.

Local and global issues in choosing applications

At the compulsory level it would be convenient to find interesting applications to be considered in the classroom. These applications could play a double role. One is to provide motivation for learning mathematics or to practice knowledge already acquired. The other is to prepare our future citizens in «their» context.

The table below sums up (in random order) various aspects that may influence our choice of applications:

LOCAL ISSUES	GLOBAL ISSUES
1. The location of the school	6. The global world
2. Social and cultural characteristics	7. Available technology
3. The time factors	8. Critical views
4. The level of learners	9. Appropriate topics
5. The skills of teachers	10. The foreseeable future

Hands-on teaching materials

Let us remember that some years ago, in April 1957, the 11th Meeting of the CIEAEM took place in Madrid, in the public school Instituto de San Isidro. The *spiritus movens* of the meeting was Pere Puig-Adam, and the main topic was «Teaching materials». Pere Puig-Adam had a strong influence on mathematics education in Spain and the publication in Spanish of the CIEAEM proceedings (Gattegno *et al.*, 1967) became an interesting source of ideas. This text benefited from the contributions of C. Gattegno, E. Castelnuovo, J.C. Nicolet, J.J. Fletcher, L. Motard, L. Campedelli, A. Bignenet, J.W. Peskett and P. Puig-Adam and most of the ideas which were described in this text are still current.

Table 1

	EXAMPLES OF LOCAL APPLICATIONS FOR TEACHING PURPOSES
1	▪ Geometrical characteristics and measurement of the school's buildings. ▪ Distances from school; times to reach the school. ▪ Geographical coordinates of the center of the school. ▪ Sources of information in the school. ▪ Food in the school and costs.
2	▪ Measurements and proportions in popular dishes. ▪ Geometry of particular buildings in town. ▪ Mathematics in folk dances. ▪ Different scales in local charts. ▪ Statistical study of minorities in local society.
3	▪ History of calendars. Local holidays. ▪ Mathematics and sports. Olympiads. World records. ▪ Art exhibits in the town. ▪ Mathematics in newspapers and magazines. ▪ Mathematics in consumer issues. Indices.
4	▪ Numbers and classical tales. ▪ Numbers in popular sayings. ▪ Mathematics and music. ▪ Alcohol rates and driving: waiting times. ▪ The cost of having a motorcycle.
5	▪ Organize visits to industries using mathematical principles. ▪ Select projects to be used for team assessment. ▪ Design a maths lab for the classroom. ▪ Organize maths activities for a weekend workshop. ▪ Collect teaching materials from everyday objects.

I would like to bring back the spirit of this old CIEAEM meeting and to defend *the need to produce good teaching materials*. With the exception of some elementary geometrical shapes, the market does not produce materials to be bought. Furthermore, it is frequently argued that hands-on material may be helpful only at the very elementary levels.

If we want to show applications and modelling procedures, we can find in our homemade materials great opportunities *to bring «real» objects into the class and to provoke an experimental research approach by modelling by means of specific materials*.

Most materials can help to promote an interdisciplinary dialogue, may help to visualize properties better... and may increase interest in the formulated problems at all levels. Many examples will be presented.

Table 2

	EXAMPLES OF GLOBAL APPLICATIONS FOR TEACHING PURPOSES
6	. Demographic issues: perspectives and extrapolations. . Insurance in travelling: accident frequencies, etc. . Mathematics in democracy. . Solidarity funds. . Ecological problems: the Kyoto agreement.
7	. Mathematics and traffic (cars, roads, gas, etc.). . Locations in the planet (GMS system). . Air traffic control: the keys to avoiding crashes. . Digital images as messages. . Implications of air conditioning in housing.
8	. Statistics on prices: imports and exports. . Visual displays from everywhere. . A visit to a car factory: sequential working. . A visit to a food factory: quality control. . Statistics about political elections: polls and votes.
9	. Codes, phones and messages. . Mathematics and genomics. . Art: paintings, sculptures, buildings. . Geometry in theme parks. . Fair division: geometry and equity.
10	. Mathematics and information: CD versus DVD, etc. . Security of data on the Internet. . Virtual museums of science and technology. . Airplanes: speeds and capacities. . Geometry and virtual 3D images.

...but do not forget the key point!

To end this presentation let us clarify something which is crucial. All kinds of applications may be helpful to make visible mathematical dimensions of our daily life on this planet. But you need to remember that among all the things you bring to the classroom there is one which is essential. There may be no chalk and there may be no materials, but at least one thing cannot be forgotten: Your enthusiasm and your eagerness with regard to mathematics, the students and your job.

Thanks for bringing all these virtues to your classes every day!... Too much is not enough!

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2

Recognising Commonalities and Reconciling Differences in Mathematics Education

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Abstract

Data from a study of the practices of research mathematicians are used to highlight an exploration of commonalities across and differences between learners in mathematics classrooms. Such commonalities and differences are, it is claimed, central to a teacher's understandings of how to address her role and responsibilities. Commonalities discussed are those between learners, across mathematics and between communities of practice. Differences between syllabus and practice, between school mathematics, mathematics in the world, and academic mathematics and, finally, between members of the different communities of students, teachers of mathematics, mathematics educators, and mathematicians are explored.

Résumé

Nous illustrerons une exploration des points communs et des différences entre apprenants dans la salle de classe à l'aide de résultats d'une étude sur les pratiques de chercheurs en mathématiques. Ces points communs et ces différences sont fondamentales pour que le professeur comprenne comment orienter son rôle et ses responsabilités. Nous discuterons des points communs entre apprenants, au travers des mathématiques et entre communautés de pratiques.

Nous explorerons les différences entre syllabus et pratiques, entre mathématiques de l'école, mathématiques du monde et mathématiques académiques et finalement entre membres des différentes communautés d'élèves, de professeurs de mathématiques, d'éducateurs en mathématiques et de mathématiciens.

Introduction

Working on the assumption that the reason we go into classrooms, whether as researchers or practitioners, or both, is because we are concerned with learning, and the learning of mathematics in particular, I focus on commonalities and differences

within classrooms as a structure to help in understanding and using different aspects of classroom practice and their outcomes, and the communities who are affected by those practices.

Theoretical Context

To situate my own perspectives on learning and, consequently, the theoretical context of this paper, I draw on two books, *Communities of Practice* by Etienne Wenger, published in 1998 by Cambridge University Press and *Learning and Awareness*, by Ference Marton and Shirley Booth, published in 1997 by Lawrence Erlbaum.

My assumption as to what matters about learning and as to the nature of knowledge, knowing, and knowers can be succinctly summarized as follows. I start with four premises:

1. We are social beings. Far from being trivially true, this fact is a central aspect of learning.
2. Knowledge is a matter of competence with respect to valued enterprises – such as singing in tune, discovering scientific facts, fixing machines, writing poetry, being convivial, growing up as a boy or a girl, and so forth.
3. Knowing is a matter of participating in the pursuit of such enterprises, that is, of active engagement in the world.
4. Meaning – our ability to experience the world and our engagement with it as meaningful – is ultimately what learning is to produce. (Wenger, 1998, p. 4)

To work with this theory of learning, accepting these premises, leads to the assertion of the following:

- Learning in the mathematics classroom is social, not individual.
- Coming to know mathematics depends upon active participation in the enterprises so valued in that community of mathematics practice that they are accepted as knowing within that community.
  The important words to stress here are «active participation». The identity of the learner is as an enquirer and the valued enterprises reflect not only the knowledge of mathematical objects and skills but also the knowledge of how mathematical enquiry is pursued. In the classroom, this involves, amongst other things, not only the potential to create knowing but also to convince oneself and other community members of the acceptability (validity) of that knowing. Sometimes the process of argument, including justification, will lead to falsification of the initial knowing, itself leading to new knowing. At other times, the process of convincing will mirror that which has been observed historically within the community of mathematicians and which has not always been without critique and challenge. The power of the community to push towards cultural conformity can, in my opinion, only be resisted and challenged if learners engage in the debate and argument necessary to construct and defend their knowing. Hence,
- The purpose of learning is to make mathematical meaning.

> This does not mean that the purpose of learning is to acquire the mathematical meaning of others but to be positioned and to position oneself as the agent of one's own learning.
> There is a potential for conflict within these statements. For example, what if an accepted practice within the school community is to work as an individual in a competitive context and to understand knowledge as an individual construct? What does this imply for learning being social?

It seems to me that where there is an implicit conflict, we must look more closely at how we understand the premises, individually and together. Under such a challenge, my perspective is to take the making of meaning as, necessarily, a social act since meaning is accorded through social agreement. Where the knowledge created and recognised through individual and competitive work is defined «objectively» this is at odds with my understanding of knowing resulting from meaning being made by learners. So, even if such practices are understood socially, they are inconsistent with my theoretical stance.

Etienne Wenger goes on to explain his term «communities of practice» as dependent upon a theory of learning which describes «being active participants in the practices of social communities and constructing identities in relation to these communities» (Wenger, 1998, p. 4) and he offers «an initial inventory» of the inter-connected components of this theory as:

> *Community (learning as belonging).*
> *Identity (learning as becoming).*
> *Practice (learning as doing).*
> *Meaning (learning as experience).* (ibid., p. 5)

From our classroom perspective, therefore, with this theoretical position, we have the following questions:

- How is the mathematical learning community constituted, how does it function and how is it sustained?
- How are the identities of pupils and teachers sustained and nurtured?
- How are learning practices experienced?
- In that process, how is meaning recognised and made?

Ference Marton and Shirley Booth add to this by asking us not to make a distinction between inner and outer, between the knower and the known but to see them as constituting an experienced whole.

> *Whatever we learn or gain knowledge about, our learning takes place as a part of our ongoing exploration of the world, our constituting the world. This means that when we are learning about something specific, we are also aware of the world around us. That which we learn about has to become a part of the world we know. The experience is, moreover, generally a mediated experience: We do not face the phenomenon as such, but the phenomenon as described by others. To an increasing degree we see the world in terms of patterns of a shared culture through a shared language.* (Marton and Booth, 1997, p. 138-139)

In the mathematics classroom, therefore, we can add two further questions:

- How is what is being learnt connected to what is known and familiar?
- In turn, how does what is being learnt inform and enhance knowledge and experience?

Out of the theoretical stance that I have constituted from these authors, through the questions that I have articulated, I draw a discussion on commonalities and differences. In doing so, I use findings from my own research to highlight how similar are the practices of research mathematicians when they are behaving like learners, that is in doing their research, to the behaviours of less mathematically sophisticated learners (Burton, 2001) and consequently how we can make use of these practices in our classrooms.

Recognising Commonalities

I find commonalities in three different sites in mathematics classrooms. They are between learners, across mathematics, and within different communities of practice and I deal with each in turn.

Commonalities between learners

There is an assertion, usually not interrogated, that learners of different ages and so-called «abilities» require different classroom activities, expectations and treatment by teachers. I want to challenge this assertion. Paul Ernest (1998) drew attention to differences between the culture of the mathematics classroom and of mathematics researchers. He cited differences between:

- Creating new knowledge and learning existing knowledge.
- Selecting the problems on which to work and having these problems imposed by teacher or text.
- Working over an extended period of time and short-term working.
- Contributing to public knowledge and learning for personal achievement.

I do not believe that these differences are a function of where the learning is taking place, school or university or industry, nor of the sophistication of the learner. I think that pointing to a difference between the creation of new knowledge and the learning of existing knowledge obscures the meaning of «new» since, taking the theoretical perspective that I have already outlined, every act of learning must, for the learner, be an act of creation, whether that learning is new in the public domain or part of the existing canon. I believe that differences such as those cited by Paul Ernest, which certainly do exist, are a product of a climate for learning created by beliefs in the so-called objectivity of mathematical knowledge and the impact that such beliefs have on classrooms. They consequently have no place within the theoretical perspective that I am adopting since I understand mathematics as an artefact of the culture in which it is created. Its observed universality, therefore, is a cultural creation and not to do with any inherent and person-independent features. That is not to deny consistency of mathematical argument, of course. But it is to assert that the starting axioms are a product of cultural choice. Different axioms have led, in the

past, to different mathematics and no doubt will do so in the future. For a familiar example of this, look at geometries.

I am maintaining that learners of mathematics differ only in sophistication of skills and knowledge but do not differ in approaches to learning. Findings from my own research highlight how similar are the practices of research mathematicians when they are behaving like learners to the behaviours of less mathematically sophisticated learners (Burton, 2001). There are commonalities that I believe can be applied from mathematicians as learners to students. These can work to the benefit of learners and teachers. In the study on which I am drawing, I interviewed 70 research mathematicians, 35 females and 35 males, about how they came to know mathematics. Here, I want to draw attention to the commonalities between the components of an epistemological model derived theoretically and published in my 1995 paper and the practices of the mathematicians described in my study. The model contained the following categories: Person- and cultural/social-relatedness, Aesthetics, Intuition/Insight, Different approaches particularly in thinking styles and Connectivities (Burton, 1995, p. 287). Let me summarise the appropriate commonalities by creating the following table to which we will return. The five model categories constitute the epistemological model I presented in (Burton, 1995). The classroom commonalities are those classroom features derived from the model categories and consequently related to them. I begin with the communalities associated with the first model category:

MODEL CATEGORIES	CLASSROOM COMMONALITIES
Person and cultural/social relatedness	Community of practices
	▪ Identity ▪ Agency ▪ Collaboration

As a result of the study, in a paper published in 1999, I described the following practices. The mathematicians acknowledged their membership of a community, sometimes particular and extremely focussed, sometimes much broader. In acknowledging such membership, they were implicitly, sometimes explicitly, recognising the discourse, the relationships, their roles and functions relative to various other members; that is, they were describing their identity. These were learning communities in ways very similar to a classroom learning community and, just as in classrooms, not all of their practices, in this case behavioural, were welcomed by their colleagues. Occasions were described, often at conferences, where aggressive and competitive behaviours unacceptable to the author of the description were observed or experienced. In this way, and many others, they made clear that theirs was the agency – they were independent thinkers, ready to challenge, question, raise connections, demonstrate their agentic role.

Despite sometimes acting in very competitive ways, nearly all of the research mathematicians were working together with someone; of the 70 mathematicians, only 3 males and 1 female claimed to work on their research problems entirely alone. This working together was called «collaborating» although I distinguished between what I

called *cooperative working*, which was usually cross-disciplinary, each team member bringing their own different expertise, and *collaboration* where they shared the same disciplinary approaches and worked together to solve the problem (Burton, 1999*a*, p. 130). Their reasons, in their words, for why they found collaboration beneficial reflected the reasons to be found in the educational literature:

Talking is a good way to get a problem done.

- *It shares the work.*
- *You benefit from the experience of others.*
- *You increase the quantity and quality of ideas.*
- *It enhances the range of skills.*
- *You get into areas that you might not have thought of going into.*
- *You learn a lot from more senior colleagues.*
- *Under the pressure of writing up, you mustn't let others down.*
- *There is someone to take over when you reach a dead-end.*
- *You have someone with whom to share «the euphoria».*
- *You feel less isolated.*
- *You can benefit from a novice/expert combination.*

(Burton, 1999*a*, p. 128)

These practices are consistent with the model of learning I have adopted. The emphasis is on the community and the ways in which membership of that community supports the identity and agency of its members who, in turn, observe functional (and dysfunctional) behaviours, and reflect on ways of influencing these. All experiences, including classroom experiences, contribute to our sense of self, our identity, and help us to locate that identity in relation to others and in relation to how we perceive our roles and responsibilities, our agency (Holland et al, 1998). For many pupils, their identity in the mathematics classroom is constructed around a sense of their own failure and, consequently, interacts with their dependency on teacher and text to provide the «game rules». In an effective classroom, pupil identity is built in relation to their agency as independent learners with the will and power to construct their knowing. Without using the term, I think Etienne Wenger gave us a lovely sense of agency in the following:

> *Words like «understanding» require some caution because they can easily reflect an implicit assumption that there is some universal standard of the knowable. In the abstract, anything can be known, and the rest is ignorance. But in a complex world in which we must find a livable identity, ignorance is never simply ignorance, and knowing is not just a matter of information. In practice, understanding is always straddling the known and the unknown in a subtle dance of the self. It is a delicate balance. Whoever we are, understanding in practice is the art of choosing what to know and what to ignore in order to proceed with our live.* (Wenger, 1998. p. 41)

The strategies that are invoked for tackling mathematicians' research problems, again, are those appropriate to learning in classrooms. Working together, that is «active participation», exploiting similarities and differences, expecting to meet blocks to understanding, but also to be rewarded with insights, are strategies for acquiring «competen-