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A Book Of

BUSINESS STATISTICS

M. Com. Part I; (Semester - II)
Course Code - 202 (B)

As Per New Syllabus, Effective from June 2019

(CBCS Pattern 2019)

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ADVANCEMENT OF KNOWLEDGE

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Statistical Thinking will one day be necessary for effective citizenship as the ability to read and write

- H.G. Wells

Preface ...

I am indeed very happy to place this book is in the hands of **M.Com. Semester-II** students. This book is written according to new prescribed syllabus (CBCS Pattern) by Pune University which comes into force from the academic year 2019.

The main purpose of the book is to provide foundation as well a comprehensive background of 'statistical tools and methods ' to beginners in simple and intersecting manner. In order to make the contents of the book easier to comprehend, I have included a requisite number of illustrations, remarks, figures, diagrams etc. To elucidate statistical concepts, Applications of Statistics in real life situations is emphasized through illustrative examples. Ample number of graded problems, are provided at the end of each chapter along with hints and answers.

While writing the book we have borne in mind that many students have not offered mathematics at XIth and XIIth std.

This book will also serve the purpose of reference book for M.B.A., C.A., B.C.A., I.C.W.A., M.P.M., classes.

I am thankful to Mr. D. K. Furia, Mr. Jignesh Furia and the staff of Nirali Prakashan for bringing out this book in short time. Anagha Medhekar, Mr. Rahul Thorat, Mrs. Anjali Muley deserve special thanks for the co-operation they have extended to us. Finally, my family deserve special thanks for their support, encouragement and tolerance.

I request colleagues teaching Statistics to offer their criticism and suggestions, for further improvement of the book.

February 2020 **Author**

Syllabus ...

1. Time Series

Introduction, Definition, Components of Time Series, The Trend, Seasonal variation, Cyclical variation, Irregular variation, Methods of estimating Trends, Moving averages (with periods 3, 4, 5), Fitting of trend line and second degree curve, Exponential smoothing, Example and problem

2. Theory of Probability Distributions : Discrete and Continuous

1.1 Random Variables, discrete random variable, continuous random Variable

1.2 Probability distribution and probability mass function (p. m .f.) of discrete random variable, Probability density function (p.d.f.) of continuous random variable

1.3 Expected value, variance and standard deviation

1.4 Marginal, Joint and Conditional distribution

1.5 Numerical Problems on finding p.m.f/p.d.f, expected value and variance

3. Standard Probability Distributions and Parameter Estimation

2.1 Binomial Distribution : p.m.f., mean and variance.

2.2 Poisson Distribution : p.m.f., mean and variance

2.3 Normal Distribution : p.m.f., mean, variance, properties

2.4 Exponential distribution : definition, mean, variance and properties

2.5 Limiting relations between these distributions

2.6 Numerical problems to calculate probabilities, mean and Variance

Parameter Estimation :

3.1 Parameter and Statistic

3.2 Unbiased estimator

3.3 Confidence interval (around unbiased estimator)

3.4 Examples and Problems on real life situations

4. Tests of Hypothesis

3.1 Hypothesis, null and alternative hypothesis, two types of errors, test statistic, critical region acceptance region, level of significance, p-value

3.2 Chi-square test for goodness of fit

3.3 Chi-square test for independence of two attributes

3.4 Small Sample Tests

(a) One sample test

(b) Two sample test

(c) Paired t – test

(d) F-test

3.5 Large sample tests for population mean and population proportion

3.1.1 Test for the mean a) one sample b) two samples

3.1.2 Test for the proportion a) one sample b) two samples

3.6 Numerical Problems

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Chapter 1...

Time Series

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1.1 Introduction

1.2 Meaning of Times Series

1.3 Components of Time Series

1.4 Analysis of Time Series

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1.6 Measurement of Trend

1.7 Measurement of Seasonal Variations

Keywords ...

Time series, Secular trend, Seasonal variations, Cyclical variations, Irregular variations, Business cycle, Additive model, Multiplicative model, Moving average, Least square method, Exponential smoothing.

Learning Objectives ...

Business forecasting needs different models appropriate to the situations. Variables which are time dependent need different techniques of analysis. Usual techniques used in descriptive statistics are not sufficient. Separation of components in time series and forecasting are two important tasks to be studied in analysis of time series.

1.1 Introduction

In the field of economics, business commerce and management forecasting is often required for several reasons.

Prices of commodities, agricultural production, mineral production, national income, prices of shares, number of passenger travelling from a certain place, revenue collection, population, volume of import and export, volume of foreign exchange, electricity consumption of a city are the examples where the changes take place every now and then. Thus prices, production, consumption are the main areas where the changes are bound to occur with respect to time. In the examples cited, above reliable forecasting needs the knowledge of the reasons behind the changes, the time epochs of changes and the magnitude of changes. The techniques and tools of forecasting are included in this chapter titled *'Time series'*. Earlier, we have studied some forecasting tools such as regression lines, fitting of curves, interpolation etc. However, those tools are useful for longterm, having steady and

smooth curve. In reality we rearely see a smooth curve; how to adjust for these temporary fluctuations is studied in this chapter. The examples given above in the field of commerce, economics are called as 'Time series'. The precise definition is discussed subsequently. Number of definitions of time series are available. Some of them are given below.

1.2 Meaning of Time Series

Definition (According to Morris Hamburg) : Time series is a series of statistical observations arranged in chronological order.

The observations in the chronological order means in the order of occurrence, taken at a regular successive intervals or points of time. The time intervals may be years, months, weeks, days, minutes and seconds also in some cases.

Following are some examples of time series.

1. Daily price of gold.
2. Weekly sales of departmental store.
3. Monthly deposits in a certain bank.
4. Yearly production of food grains in a certain country.
5. Daily record of maximum temperature in a city.
6. Hourly bacterial count in certain culture at laboratory.
7. Population of country at census years.

According to Spiegel, mathematically a time series is defined by the values $Y_1, Y_2, \ldots, Y_n, \ldots$ of the variable Y at times $t_1, t_2, \ldots, t_n, \ldots$. Thus, time series is a function of time i.e. $Y = F(t)$. In other words, in time series time plays the role of an independent variable and Y (t) is dependent variable. We denote time series by Y (t) or Y_t. In the form of function the time series may be written as follows :

t	t_1	t_2	…	t_n	……
Y_t	Y_1	Y_2	…	Y_n	……

The time points $t_1, t_2, t_3, \ldots$ are equidistant. The analysis of time series is important from many aspects. We plot t on X-axis and Y_t on Y-axis. Consider, the example of population of India in census years.

Year (t)	1901	1911	1921	1931	1941	1951	1961	1971	1981	1991	2001	2011
Population (Cores) Y_t	23.84	25.21	25.13	27.9	31.87	36.11	43.92	54.82	63.33	84.39	102.70	121.02

We observe that the population is measured during census years with intervals of ten years. Hence, the figures are arranged chronologically, that is in order of time. If they are not ordered according to time, then they would not be able to provide information about the pattern of variation. If you carefully observe the above time series, you will notice that the population has an increasing trend. Now have a look at the following graph.

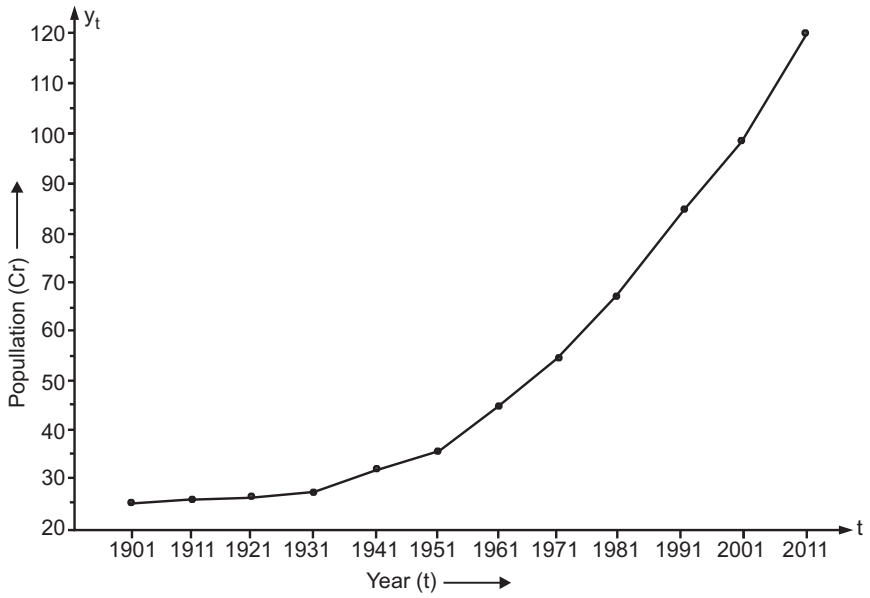


Fig. 1.1

The following things are worth noticing.

(i) India's population is steadily increasing since 1921.

(ii) The rate of population increase accelerates further since 1951, that is, after independence.

Government wants to predict the population of India for future, say for the year 2010; as important policies are to be designed on the basis of such estimates. Time series analysis helps in doing so.

A time series is a summary of past information. Assuming that the 'past' serves a guide to the 'future', time series analysis helps in detecting the underlying patterns and projecting them into future.

Some authors describe the time series data represented graphically as **historigram.**

The nature of time series graph is usually not smooth and not monotonic, it is zig-zag or haphazard. The critical study reveals that these fluctuations are not totally haphazard, however some part is systematic and the only counter part is haphazard in nature.

1.3 Components of Time Series

Time series Y_t is composed of four factors or components viz. (1) Trend, (2) Seasonal variations, (3) Cyclical variations, (4) Irregular variations. These factors cause fluctuations in the values of Y_t. In other words the fluctuations in time series are classified into the above four patterns or categories. A time series may have some or all the components present in it.

1. Trend or Secular Trend (T) :

The trend is the smooth, regular long term movement in time series. It is the general tendency of data. The time series oscillates around trend. The trend may be to go upward to go downword or to remain stagnant.

For example, the yearly population, yearly agricultural production, prices etc. show an upward trend. On the other hand cost of electronic goods, number of illiterates, yearly birth rates etc. show downward trend. Yearly rainfall, daily temperature atmospheric pressure at a certain place, monthly electricity consumption of a family are the examples of constant trend. The trend may be linear or non-linear in nature.

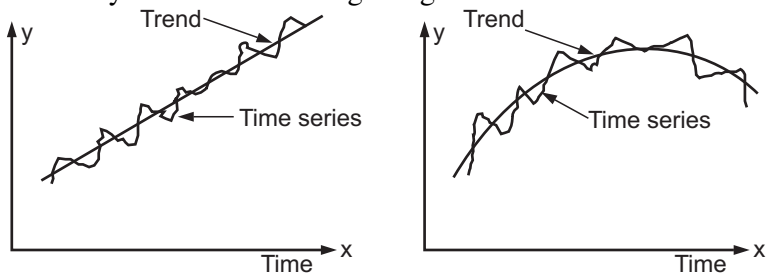
Remarks :

(i) Trend is also called as secular trend. The word secular is derived from the Latin word saeculum which means generation or age.

(ii) Trend is due to the reasons of following nature, changes in population, technological developments, changes in economy, changes in habits and tastes of people.

(iii) Trend is a long period movement however, the period cannot be precisely defined. For example, regarding price of gold, agricultural production the long period cannot be few weeks or 2, 3 years. It may be observed over a period of 10 to 15 years even more than that. Long term period may change from series to series.

(iv) Trend is mostly monotonic although original time series is not.



(a) Linear trend **(b) Non-linear trend**

Fig. 1.2

In linear trend the values of Y can be approximated by a straight line. This indicates that the rate at 'which the time series values increase or decrease is constant. On the other hand, in non-linear trend, the growth rate is different over different sectors of time. Linear trend is commonly used in business and economics. Non-linear trend is used in the study of population birth rates etc.

(v) Apart from the long-term growth component, there are some short-term periodic rhythmic variations. These variations disturb the smoothness and monotony.

(vi) Trend is useful for two reasons :

(a) It facilitates the comparison of two time series.

(b) It helps to extrapolate.

2. Seasonal Variations (s) :

Seasonal variations are the fluctuations in a time series which repeat regularly every year or some specific period of time.

For example, sales of umbrellas and raincoats is the highest in rainy season; sales of wollen garments attains its peak in winter, sales of luxury items and jewellary is high during festivals. Even the bank deposits and bank clearings are affected by seasonal swings. Here, the 'seasons' may be taken as weeks. Similarly, traffic is maximum during rush hours; so 'seasons' are hours in this case. The "seasons' may be seconds in bacterial population growth.

The seasonal variations may be either due to natural forces such as festivals climatic conditions or due to customs, fashions or habits of the people. These factors operate in a regular and periodic manner where the period of recurrence is generally one year. Following graph shows seasonal variations in the sales of umbrellas (Y) from a store.

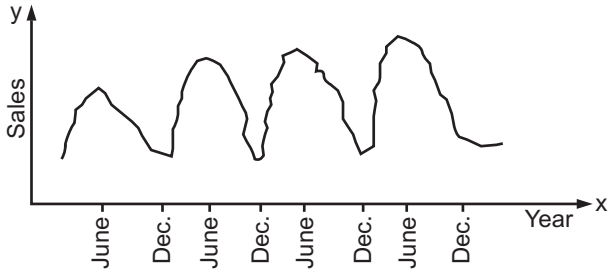


Fig. 1.3

Note that the sales attain maximum in the month of June every year.

Remark : The amplitude may differ from cycle-to-cycle.

The study of seasonal variations is of prime importance in many time series. Particularly, when the trend is stagnant, seasonal variations become predominant component.

Seasonal variations are extremely useful in marketing and business field in many ways.

For example, during summer sales of fans, coolers, refrigerator, cold-drinks, ice-creams increase considerably and reach the peak. Business man has to take care of inventory for seasonal peaks, he has to employ adequate number of salesmen, he has to schedule purchases and sales, he has to arrange for clearance sale, he has to advertise, and give discount on prices for off seasons, he has to arrange for additional finances during seasons. Similarly, bank managers has to arrange for proper cash flow during the beginning of month, festivals.

3. Cyclical variations (c) :

Cyclical variations in a time series are the fluctuations which repeat over a time period of more than one year. The cyclical variations may not be necessarily uniformly periodic. The amplitude of variation also changes from cycle-to-cycle. That is, the ups and downs may occur at different intervals of time. These fluctuations are typically observed in business. Boom in business is followed by depression and vice-versa. The period of a 'Cycle' is about 7 to 10 years. There are four phases in a cycle. (i) prosperity (boom), (ii) recession, (iii) depression, (iv) recovery. Fig. 1.4 depicts the cyclical variations comprising of these four phases.

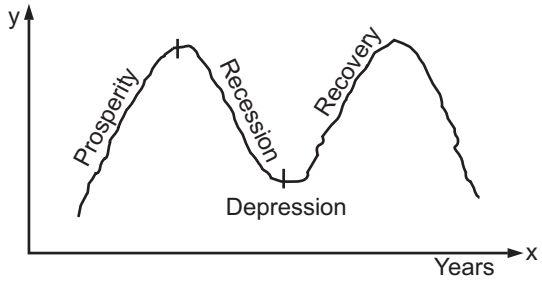


Fig. 1.4

Time series relating to productions, wages bank deposits etc. exhibit cyclical variations in a prominant way.

Business cycle is the main cause behind the cyclical variations. Imbalances in economy, import and export facilities and policies, availability of loans, inflation, over-development, decreasing efficiency of personnel, automation, business competitions are some reasons which result into business cycle.

Difference between 'seasonal variations' and 'cyclical variations' :

(i) Reason : Seasonal variations are evident due to seasons, festivals, customs and needs of people. Whereas cyclical variations exist due to business cycle.

(ii) Period : Seasonal variations are up and down swings of shorter period less than one year. Cyclical variations are of longer period 5 to 10 years or higher in some series. Seasonal variations occur more or less regularly at a certain time period. Cyclical variations may differ in the period from cycle-to-cycle. They do not occur at a specific period. Both seasonal variations and cyclical variations do not have successive cycles of uniform amplitude.

(iii) Comparatively intensity of seasonal variations less than that of cyclical variation.

4. Irregular Variations (I) :

Irregular variations are unpredictable and are the results of unforeseen forces or abnormal events. These variation therefore do not follow any pattern; neither in their magnitude nor in the time of their occurrence. These variations are generally caused by calamities such as earthquakes, famines floods, epidemics or abnormalities such as war, stikes, lockouts etc. Hence, they are also called as 'episodic fluctuation'.

Irregural variations occur randomly hence they are also called as 'random variations'. These variations are mixed-up with seasonal and cyclical variations. The reasons behind irregular variations are non-recurring hence is difficult to isolate such variations and analyse. Sometimes, the irregular variations are minor in magnitude while in others they are so large due to abnormal reasons, that they produce cyclical variations.

1.4 Analysis of Time Series

Time series analysis, in simple words means, study of time series. The purpose of time series analysis is two fold.

(i) Identifying the above four components which cause variations in the variable, and

(ii) Isolating, studying and measuring each of them independently.

Models for Time Series :

In analysis, it is required to know how the components interact and give the joint effect which is done with the help of models. Generally, two types of models are used to describe a time series Y_t.

1. Additive Model :

Let Y denote the original time series and T, S, C, I the four components. Under additive model,

$$Y = T + S + C + I$$

This model assumes that the original time series is mere sum of the four components. In turn, it assumes that the four components have no interaction and act independently. Whenever, the changes are by a constant amount, additive model is used.

Illustration : Suppose a time series has components Trend (T) = 300, Seasonal variation (S) = 90, Cyclical variation (C) = 20 and Irregular component (I) = – 15 at fixed point t then under additive model

$$Y_t = T + S + C + I = 300 + 90 + 20 - 15 = 395$$

The assumption in the additive model is that the components are non-interactive or independent is non-realistic. In order to overcome these limitations, multiplicative model is popularly used.

2. Multiplicative Model :

Under this model, the original time series Y is assumed to be the effect of the four components working interactively. Hence,

$$Y = T \cdot S \cdot C \cdot I$$

In Economics, wherever the changes are by a constant rate, multiplicative model is used.

Using either of these models, the components are eliminated one-by-one. This elimination helps in isolating the factors from one another and measuring them independently.

There are other types of models also which involve some components additive and some components multiplicative.

For example, $Y = T \times C + S \times I$, $Y = T \times C \times S + I$.

These are beyond the scope of this book.

Illustration : Suppose a time series with multiplicative model has components Trend (T) = 500, Seasonal variation (S) = 1.3, Cyclical variation (C) = 1.2 and Irregular variation (I) = 0.9 at a certain time point t then $Y_t = T \times S \times C \times I = 500 \times 1.3 \times 1.2 \times 0.9 = 702$.

Note :

1. In additive model the components are expressed in terms of actual absolute (not relative) values and Y_t is the algebraic sum of them. However, in multiplicative model the components except trend are interms of relative values or rates or percentage of trend. The trend is expressed in terms of actual value. Y_t in this case is product of all components.
2. Under the additive model the algebraic sum of seasonal variations in a year is zero per year ($\sum S = 0$, per year), that of cyclical variation per cycle is zero ($\sum C = 0$, per year). In the long run sum of irregular variations ($\sum I = 0$).
3. Multiplicative model can be converted to additive if we take logarithms

$$Y = T \cdot S \cdot C \cdot I$$
$$\log Y = \log T + \log S + \log C + \log I$$

1.5 Utility of Time Series Analysis

Time series analysis is of paramount interest in various disciplines such as economics, business, social sciences. Its uses are discussed below :

(i) Past behaviour : It enables to describe the past behaviour of the variables. Time series analysis reveals the forces working behind the series such as technological and economical developments, changes in import and export policies.

(ii) Forecasting : Forecasting is one of the important use of analysis of time series. The forecasting in business plays an important role in planning decision-making, inventory, scheduling of purchases and sales etc.

Isolating and measuring the effects of various components help the investigator to forecast the value of variable in future with fairly good reliability.

(iii) Comparison : Time series analysis facilitates the comparison between the two related time series.

For example,

(a) Prices of gold and prices of shares,

(b) National income and cost of living indices.

Comparison between actual and expected performance can be made comparison between two similar time series at two different places.

1.6 Measurement of Trend

Trend can be estimated by several methods of which we consider the following.

(1) Graphical Method (2) Method of Moving Averages (3) Least Squares Method (4) Exponential Smoothing Method.

(i) Graphical Method (Free hand curve) : This method of estimating trend is also known as "free hand curve fitting" method. It consists of (i) plotting the time series data by taking 'time' on X-axis and the variable of interest on Y-axis and (ii) drawing a smooth free hand curve which would exhibit the long term tendency in the series. [See Fig. 1.5]

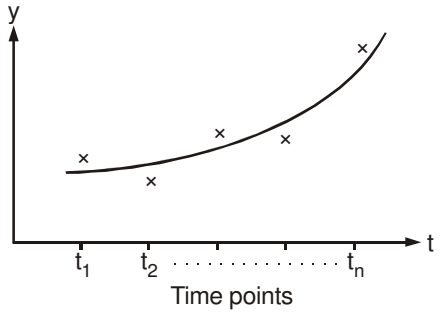


Fig. 1.5

While drawing the curve the following points should be taken care of :

(i) It should be a smooth curve.

(ii) It should pass through as many as points it can.

(iii) Almost equal number of points should lie on either side of the curve. So that the curve is as close as possible to the points.

(iv) The sum of vertical deviations from the curve should be approximately zero.

(v) Almost equal number of cycles should be on the either side of the curve.

(vi) If the curve bisects the cycles then the area between cycles and curve on both sides should approximately same.

(vii) Fluctuations of smaller magnitude be ignored.

Navie Fore Cast : Extending the curve drawn, we can forecast the values in future.

This fore cast is very approximate and no exact mathematical model is used hence it is considered as naïve fore cast.

Merits of Graphical Method :

(i) It is the simplest and crudest method of estimating trend.

(ii) It saves time and does not need any mathematical calculations.

Demerits of Graphical Method :

(i) It is rather subjective method. Different persons can get different curves for the same time series.

(ii) Due to this subjectivity, it is dangerous to use this method for forecasting purposes.

(2) Method of Moving Averages : Measurement of trend is possible only when fluctuations arising due to other three components of time series (viz. seasonal, cyclical and irregular) are smoothened out. This is nicely achieved in the method of moving averages. It consists of obtaining arithmetic means of successive overlapping values. The method proceeds as follows. Suppose the following is a time series.

Time	t_1	t_2	…	t_n
Y	Y_1	Y_2	…	Y_n

Suppose, we want to find moving averages of period 'm' say. This 'm' called period of moving average is generally taken equal to the time interval for which two successive maxima of y are seen.

The series of moving averages is calculated as follows :

$$\text{First M.A.} = \frac{Y_1 + Y_2 + \ldots\ldots\ldots + Y_m}{m}$$

$$\text{Second M.A.} = \frac{Y_2 + Y_3 + \ldots\ldots\ldots + Y_{m+1}}{m}$$

$$\text{Third M.A.} = \frac{Y_3 + Y_4 + \ldots\ldots\ldots + Y_{m+2}}{m} \quad \text{and so on.}$$

Two cases arise.

(i) m odd : In this situation moving average is placed against the mid value of the time interval it covers.

For example, if m = 5, the moving average will be placed in front of the third value in the group.

(ii) m even : When m is even, the mid value does not coincide with the original time series, in order to make it coincide with the time series, moving averages of order two are again taken. This process is called centering. (See Ex. 1.2).

Solved Examples

Example 1.1 : The following data give the sales (in '000 ₹) of a company for the years 2010-2019.

Year (t)	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Sales (y)	50	82	65	86	70	52	90	65	87	43

Calculate : (i) 3 yearly moving averages. (ii) 5 yearly moving averages.

(iii) Plot the original time series alongwith the 3 yearly and 5 yearly moving averages.

Solution :

t	y	3 yearly total	3 yearly m.a.	5 yearly total	5 yearly m.a.
2010	50	–	–	–	–
2011	82	197	65.67	–	–
2012	65	233	77.67	353	70.6
2013	86	221	73.67	355	71.0
2014	70	208	69.33	363	72.6
2015	52	212	70.67	363	72.6
2016	90	207	60.00	264	72.8
2017	65	242	80.67	337	67.4
2018	87	195	65.00	–	–
2019	43	–	–	–	–

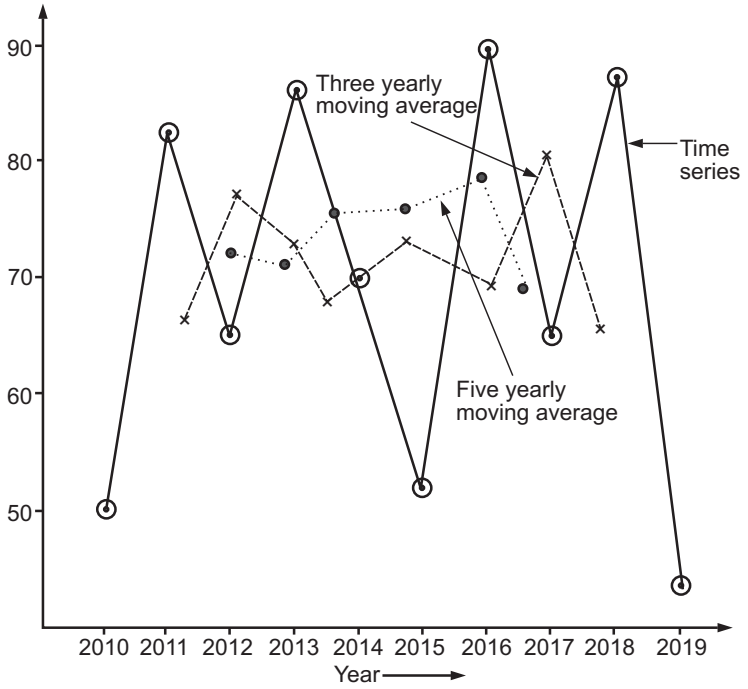


Fig. 1.6

Example 1.2 : Following data relate to the average retail price ₹ per kg of ground nut during four quarters of 5 years 2015-2019.

Year	Quarters			
	I	II	III	IV
2015	40	35	38	40
2016	42	37	39	38
2017	41	35	38	42
2018	45	36	36	41
2019	44	38	38	42

Obtain 4 point moving average.

Solution : The period of moving average is even, hence it needs to be centred. Thus in the beginning we find 4 point moving average and then 2 point average of these averages. It will ensure centering.

Calculation of moving averages

Year	Quarter	Price Y_t	4-point moving total	4-point moving average	2-point moving total	Centred moving average
2015	I	40				
	II	35				
	III	38	153	38.25	77	38.50
	IV	40	155	38.75	78	39.00
2016	I	42	157	39.25	78.75	39.38
	II	37	158	39.50	78.50	39.25
	III	39	156	39.00	77.75	38.88
	IV	38	155	38.75	77	38.50
2017	I	41	153	38.25	76.25	38.13
	II	35	152	38.00	77	38.50
	III	38	156	39.00	79	39.50
	IV	42	160	40.00	80.25	40.13
2018	I	45	161	40.25	80	40.00
	II	36	159	39.75	79.25	39.63
	III	36	158	39.50	78.75	39.38
	IV	41	157	39.25	79	39.50
2019	I	44	159	39.75	80	40
	II	38	161	40.25	80.75	40.38
	III	38	162	40.50		
	IV	42				

Remark : The 4 point centred moving average is equivalent to a weighted moving average of period 5 with weight 1, 2, 2, 2, 1.

The first centred moving average = 38.50. The five point weighted moving average with weights 1, 2, 2, 2, 1 is

$$\frac{40 + (2 \times 35) + (2 \times 38) + (2 \times 40) + 42}{8} = 38.5$$

Note that first M.A. $= \frac{Y_1 + Y_2 + Y_3 + Y_4}{4} = \bar{Y}_1$

Second M.A. $= \frac{Y_2 + Y_3 + Y_4 + Y_5}{4} = \bar{Y}_2$

Centred total $= \bar{Y}_1 + \bar{Y}_2$

$$\text{Centred M.A.} = \frac{\bar{Y}_1 + \bar{Y}_2}{2} = \frac{1}{2}\left[\frac{Y_1 + Y_2 + Y_3 + Y_4}{4} + \frac{Y_2 + Y_3 + Y_4 + Y_5}{4}\right]$$

$$= \frac{1}{8}\,[Y_1 + 2Y_2 + 2Y_3 + 2Y_4 + Y_5]$$

= Weighted average of Y_1, Y_2, Y_3, Y_4, Y_5 with weights 1, 2, 2, 2, 1 respectively.

Merits and Demerits of Moving Average Method :

Merits :

(i) The method is simple and easy to work with.

(ii) The method is not subjective.

(iii) A proper choice of period can remove seasonal, cyclical fluctuations completely and irregular fluctuations to some extent.

(iv) The method is flexible in the sense that if few more observations are added to the series, the moving averages obtained earlier remain the same.

Demerits :

(i) The method can not provide trend values for some initial and last points of time.

(ii) It can not be used for forecasting purposes.

(iii) The moving average method estimates the trend fairly well only if the trend is linear, and the ups and downs in the time series are periodic.

(3) Least Squares Method : The method of least square is an analytical method used to estimate the trend values. The relationship between the variable 'Y' and time 't' is assumed according to a mathematical function.

For example, (i) $Y = a + bt$ (Linear trend)

(ii) $Y = a + bt + ct^2$ (Parabolic trend)

(iii) $Y = ab^t$ (Exponential trend)

or $\log Y = \log a + t \log b$ (Log-linear trend)